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On equivalence question between the Ditzian-Totik modulus of smoothness and an usual periodic modulus of smoothness

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Let $f \in C[-1,1]$ and $g(t) = f(\cos t)$, $t \in \mathbf{R}$. It is well-known that $\omega_1^{\varphi}(f;\delta) \sim \omega_1(g|_{[0,\pi]};\delta)$, $0 \le \delta \le \delta_0$, where

$$\varphi(x) = \sqrt{1-x^2} \quad , \quad x \in [-1,1] \quad \text{and} \quad \omega_1^{\varphi}(f;\delta)$$

represents the Ditzian-Totik uniform modulus of continuity and $\omega_1(g|_{[0,\pi]};\delta)$ represents the usual uniform modulus of continuity of g on $[0,\pi]$ (see [2]).

Problem. *Find a function $F \in C_{2\pi}$, depending on f and $n \in \mathbf{N}$, such that*

$$\omega_n^{\varphi}(f;\delta) \sim \omega_m(F|_{[0,2\pi]};\delta), \quad 0 \le \delta \le \delta_0,$$

where $\omega_n^{\varphi}(f;\delta)$ represents the n-th order Ditzian-Totik modulus of smoothness and possibly (but not necessarily!) $n = m$.

Reason. *From computational point of view,* $\omega_m(F|_{[0,2\pi]};\delta)$ *is much more accessible than* $\omega_n^\varphi(f;\delta)$.

Remark. *Obviously* $F(t) \not\equiv f(\cos t)$ *because if* $F \equiv f \circ \cos$ *then it is known that*

$$\omega_m(F|_{[0,2\pi]};\delta) \sim w_m^\varphi(f;\delta), \quad 0 \le \delta \le \delta_0,$$

where $w_m^\varphi(f;\delta)$ *represents the Felten's algebraic modulus of smoothness (see [1]).*

References

[1] M. Felten, *Characterization of best algebraic approximation by an algebraic modulus of smoothness*, J. Approx. Theory **89** (1997), 1-25.

[2] G. Mastroianni and P. Vertesi, *Approximation by some operators using certain new moduli of continuity*, Suppl. Rend. Math. Palermo ser.II, Nr.**33** (1993), 387-397.