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The second order modulus again: some still (?) open problems

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In our talk we will make one further attempt to convince the audience that they neither fully understand the classical Bernstein operators B_n nor ω_2, the second order modulus of continuity. Although a lot of progress was achieved over the last 15 years, say, in the work of Cao, Ditzian, Gavrea, Kacsó, Knoop, Păltănea, Totik, Xin-long Zhou, and others (and maybe also by myself), a lot of problems remain to be solved.

In our presentation we will compile some of these. To give an example, here we only mention

Problem N (first given explicitly at Brno 8)

By $\mathrm{Lip}^*(\alpha;[0,1])$, $0<\alpha\leq 2$, *we denote a Lipschitz class with respect to* <u>*second*</u> *order differences.*

In papers by Cottin, Gonska and Kovacheva (3, 4) the following was proved:

$$f \in \mathrm{Lip}^*_M(\alpha; [0,1]) \Longrightarrow B_n f \in \mathrm{Lip}^*_{4.5\cdot M}(\alpha; [0,1]).$$

Later work of Adell, Gavrea, Pérez-Palomares and Ding-xuan Zhou showed that the constant 4.5 *can be improved (remarkably !).*

The problem is: What are the best possible constant $c(\alpha)$ *such that*

$$f \in \mathrm{Lip}^*_M(\alpha; [0,1]) \Longrightarrow B_n f \in \mathrm{Lip}^*_{c(\alpha)\cdot M}(\alpha; [0,1])?$$

Of course, we include the possibility to have $c(\alpha) \cdot M$ *replaced by* $c(\alpha, M)$.