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On the operator of Bleimann, Butzer and Hahn

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The classical Bernstein operators are defined by

$$B_n(h,t) := \sum_{k=0}^{n} p_{n,k}(t)\, h\left(\frac{k}{n}\right),\ t \in [0,1],\ n = 0,1,\ldots,$$

where

$$p_{n,k}(t) = \binom{n}{k} t^k (1-t)^{n-k} \quad , \quad h \in \mathbf{R}^{[0,1]}.$$

The linear operators L_n defined by

$$L_n(f,x) := \frac{1}{(1+x)^n} \sum_{k=0}^{n} \binom{n}{k} x^k f\left(\frac{k}{n+1-k}\right),$$
$$x \in [0,\infty),\ n = 0,1,\ldots,$$

where $f \in \mathbf{R}^{[0,\infty)}$, were introduced in 1980 by Bleimann, Butzer and Hahn [4] to approximate continuous functions on the positive semi-axis and has been investigated by several authors (cf. references).

We have proved the following theorem

Theorem 1. *Let* $a > 1$. *If* $f \in C[0,\infty)$, $f(x) = O(a^x)$, $x \to \infty$, *then*

$$L_n(f;x) \to f(x) \Longleftrightarrow x \in \left[0, \frac{1}{a-1}\right).$$

Prove or disprove

Theorem 2. *If* $b > 0$, $f \in C[0,\infty)$ *such that*

$$L_n(f;x) \to f(x) \Longleftrightarrow x \in [0,b)\,,$$

then $f \in O(\left(1+\frac{1}{b}\right)^x)$, $x \to \infty$.

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