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On Hermite–Lagrange series

Neculae Vornicescu

Let $x_1 < x_2 < \ldots < x_m$, $m > 2$, be a system of fixed nodes. We use the notation

$$u_m(x) = (x - x_1)(x - x_2) \cdots (x - x_m).$$

We deal with the problem to expand a function $f \in C^\infty[x_1, x_m]$ in a series of the form

$$\sum_{n=0}^{\infty} u_m^n(x)(c_{n,1} + c_{n,2}(x - x_1) + \cdots + c_{n,m}(x - x_1) \cdots (x - x_{m-1})).$$

The partial sums

$$S_n(x) = \sum_{k=0}^{n} u_m^k(x)(c_{k,1} + c_{k,2}(x - x_1) + \cdots$$

$$\cdots + c_{k,m}(x - x_1) \cdots (x - x_{m-1})),$$

are Hermite polynomials verifying:

$$S_n^{(k)}(x_i) = f^{(k)}(x_i), \qquad i = \overline{1, m},\ k = \overline{0, n}.$$

The main result is an integral representation of the coefficients $c_{n,i}$.

An improvement of the known result in the case $m = 2$ is obtained.

References

[1] G. Berger and M. Tasche, Hermite–Lagrange interpolation and Schur's Expansion of $\sin \pi x$, J. Approx. Theory **53** (1988), 17–25.

[2] G. Ramharter, A remark on Hermite-Lagrange Interpolation, J. Approx. Theor. **66** (1991), 109-113.