Georgian Mathematical Journal
Volume 10 (2003), Number 1, 37–43

BRANCHED COVERINGS AND MINIMAL FREE RESOLUTION FOR INFINITE-DIMENSIONAL COMPLEX SPACES

E. BALLICO

Abstract. We consider the vanishing problem for higher cohomology groups on certain infinite-dimensional complex spaces: good branched coverings of suitable projective spaces and subvarieties with a finite free resolution in a projective space $\mathbf{P}(V)$ (e.g. complete intersections or cones over finite-dimensional projective spaces). In the former case we obtain the vanishing result for H^1. In the latter case the corresponding results are only conditional for sheaf cohomology because we do not have the corresponding vanishing theorem for $\mathbf{P}(V)$.

2000 Mathematics Subject Classification: 32K05, 32L10.
Key words and phrases: Infinite-dimensional projective space, complex Banach manifold, branched covering, vanishing of cohomology.

1. Minimal Free Resolution

Recently, L. Lempert (see [7], [8], [6], and the references therein) gave a new life to infinite-dimensional holomorphy. The aim of this paper is to give an algebraic set-up in which these general results may be applied. We found two different set-ups and to each set-up is devoted one of the two sections of this paper. For general results on analytic sets in locally convex spaces and the projectivization of a Banach space, see [9] and [10].

In the second section we will consider nice branched coverings $f : Y \to M$ of a complex manifold M. For certain particular ramified coverings of an infinite-dimensional projective space and a fine study of their Dolbeaut cohomology, see [6, Ch. 4].

In the first section of this paper we consider finite codimensional closed analytic subspaces, Y, of a nice projective space $\mathbf{P}(V)$ such that the ideal sheaf $\mathcal{I}_Y$ of Y in $\mathbf{P}(V)$ admits a minimal free resolution of finite length. Since the results on Dolbeaut or algebraic cohomology for $\mathbf{P}(V)$ are proved in various degree of generality concerning the locally convex space V and the corresponding results for sheaf cohomology are just conjectural, we will just take as axiom that V satisfies the conditions needed to have [7, Theorem 7.3 (i)] for sheaf cohomology, not just Dolbeaut cohomology.

Definition 1. Let V be an infinite-dimensional complex vector space equipped with a locally convex Hausdorff topology. We will say that V has Property (A) or that it satisfies Condition (A) if every holomorphic line bundle on $\mathbf{P}(V)$ is a multiple of the hyperplane line bundle $\mathcal{O}_{\mathbf{P}(V)}(1)$ and $H^i(\mathbf{P}(V), L) = 0$ for every

ISSN 1072-947X / \$8.00 /

holomorphic line bundle L on $\mathbf{P}(V)$ and every integer $i > 0$. We will say that V has Property (B) or that it satisfies Condition (B) if $H^i(\mathbf{P}(V), L) = 0$ for every holomorphic line bundle L on $\mathbf{P}(V)$ and every integer $i > 0$. We will say that V has Property (C) or that it satisfies Condition (C) if $H^i(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(t)) = 0$ for all integers t, i such that $i > 0$.

Unless otherwise stated, we will always consider sheaf cohomology groups in Definition 1 and elsewhere in this paper. When the base space is paracompact, these cohomology groups are isomorphic to the Čech cohomology groups of the same sheaf [4, Corollary at p. 227]. We recall that for every Banach space V the projective space $\mathbf{P}(V)$ is metrizable. For instance, this can be checked defining on $\mathbf{P}(V)$ the so-called Fubini–Study metric or, calling S the unit sphere of V, using that the restriction to S of the quotient map $\pi : V\backslash\{0\} \to \mathbf{P}(V)$ has compact fibers to define a metric on $\mathbf{P}(V)$ from the norm metric on S. Thus by a theorem of Nagata and Smirnov $\mathbf{P}(V)$ is paracompact. If V has no continuous norm, then $\mathbf{P}(V)$ is not even a regular topological space [7, p. 507].

Definition 2. Let V be an infinite-dimensional complex vector space satisfying Condition (C) and Y a closed analytic subspace of $\mathbf{P}(V)$. We will say that Y has finite cohomological codimension in $\mathbf{P}(V)$ if the ideal sheaf $\mathcal{I}_Y$ of Y in $\mathbf{P}(V)$ has a finite free resolution, i.e., if there are an integer $r \geq 0$ and sheaves E_i, $0 \leq i \leq r$, on $\mathbf{P}(V)$ with each E_i a direct sum of finitely many line bundles $\mathcal{O}_{\mathbf{P}(V)}(t)$, $t \in \mathbf{Z}$, such that there there is an exact sequence

$$0 \to E_r \to E_{r-1} \to \cdots \to E_1 \to E_0 \to \mathcal{I}_Y \to 0. \tag{1}$$

Every map $E_i \to E_{i-1}$ in (1) is given by a matrix of continuous homogeneous polynomials. We will say that (1) is minimal if for every integer i with $1 \leq i \leq r$ the matrix associated to the map $E_i \to E_{i-1}$ has no entry which is a non-zero constant.

As in the case of a finite-dimensional complex projective space a minimal free resolution is essentially unique, i.e. for every integer the integer r and the isomorphism classes of all vector bundles E_i, $0 \leq i \leq r$, are uniquely determined by Y. Furthermore, from any finite free resolution it is easy to obtain a minimal free resolution, just deleting certain direct factors of the bundles E_i. The big difference is that in the infinite-dimensional case we were forced to assume the existence of a free resolution of $\mathcal{I}_Y$ (and we believe that this is a restrictive assumption), while every closed analytic subspace of $\mathbf{P}^n$ has a free resolution because it is an algebraic subscheme (Chow theorem) and we may apply Hilbert syzygy Theorem. Only the Chow theorem is available for an arbitrary finite-codimensional closed analytic subset Y of the projectivization of an infinite-dimensional Banach space [10, III.3.2.1].

Remark 1. We only know two classes of subvarieties $Y \subset \mathbf{P}(V)$ with finite free resolution. The first class is given by the complete intersections of $r+1$ continuous homogeneous polynomials. In this case a minimal free resolution of $\mathcal{I}_Y$ is the Koszul complex of $r+1$ forms. Here is the second class. Take $V = A \oplus W$ (topological decomposition) with A finite-dimensional and let

$Z \subseteq \mathbf{P}(A)$ be any projective variety. Let $Y \subseteq \mathbf{P}(V)$ be the cone with vertex $\mathbf{P}(W)$ and base Z. Since $\dim(A)$ is finite, Z has a minimal free resolution as a subscheme of $\mathbf{P}(A)$. Every homogeneous form on A induces a continuous homogeneous form on V not depending on the variable W. Hence a minimal free resolution of Z in $\mathbf{P}(V)$ induces a minimal free resolution of Y in $\mathbf{P}(V)$ with the same numerical invariants (the integer r, ranks of the bundles E_i and degrees of their rank one factors).

Theorem 1. *Let V be an infinite-dimensional complex vector space satisfying Condition (C) and Y a closed analytic subspace of $\mathbf{P}(V)$ with finite cohomological codimension. Then $H^i(\mathbf{P}(V), \mathcal{I}_Y(t)) = 0$ and $H^i(Y, \mathcal{O}_Y(t)) = 0$ for all integers t, i such that $i > 0$.*

Proof. Fix a finite free resolution (1) of $\mathcal{I}_Y$ and twist it with $\mathcal{O}_{\mathbf{P}(V)}(t)$. By Condition (C) we have $H^i(\mathbf{P}(V), E_j(t)) = 0$ for $0 \leq j \leq r$ and every integer $i > 0$. Hence the vanishing of $H^i(\mathbf{P}(V), \mathcal{I}_Y(t))$ follows a splitting of (1) into short exact sequences. From the exact sequence

$$0 \to \mathcal{I}_Y(t) \to \mathcal{O}_{\mathbf{P}(V)}(t) \to \mathcal{O}_Y(t) \to 0 \tag{2}$$

and the vanishing of $H^i(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(t))$ (Condition (C)) for every $i > 0$ we obtain $H^i(Y, \mathcal{O}_Y(t)) = H^{i+1}(\mathbf{P}(V), \mathcal{I}_Y(t)) = 0$. □

Corollary 1. *Let V be an infinite-dimensional complex vector space satisfying Condition (C) and Y a closed analytic subspace of $\mathbf{P}(V)$ with finite cohomological codimension. Assume $\mathbf{P}(V)$ paracompact. Then the topological and the holomorphic classification of line bundles on Y are the same.*

Proof. The sheaf cohomology group $H^1(Y, \mathcal{O}_Y^*)$ classifies isomorphism classes of all line bundles on any reduced complex space Y. Thus $H^1(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}^*) \cong \mathbf{Z}$. From the exponential sequence

$$0 \to \mathbf{Z} \to \mathcal{O}_{\mathbf{P}(V)} \to \mathcal{O}_{\mathbf{P}(V)}^* \to 1 \tag{3}$$

and Theorem 1 we obtain $H^(Y, \mathcal{O}_Y^*) \cong H^2(Y, \mathbf{Z})$. The corresponding exponential exact sequence for topological line bundles and the paracompactness assumption imply that $H^2(Y, \mathbf{Z})$ parametrizes equivalence classes of topological line bundles on Y. Since these isomorphisms are compatible with the map associating the topological structure to any holomorphic line bundle, we conclude that the assertion is valid. □

2. Branched Coverings

Definition 3. Let Y be a connected complex analytic space, M a connected complex manifold and $f : Y \to M$ a holomorphic map. We say that f is a c-flat finite covering if it is a proper map with finite fibers and $f_*(\mathcal{O}_Y)$ is a locally free $\mathcal{O}_M$-sheaf with finite rank. The rank of $f_*(\mathcal{O}_Y)$ is called the degree $\deg(f)$ of f.

Let $f : Y \to M$ be a c-flat finite covering. The map f is surjective. Every germ of the holomorphic map on M induces a germ of the holomorphic map on Y and the associated inclusion $\mathcal{O}_M \to f_*(\mathcal{O}_Y)$ has as cokernel a locally free $\mathcal{O}_Y$-sheaf of rank $\deg(f) - 1$.

Example 1. Let V be an infinite-dimensional Banach space such that $\mathbf{P}(V)$ is localizing ([7, p. 509]), Y a connected analytic space and $f : Y \to \mathbf{P}(V)$ a c-flat finite covering of degree d. By [7, Theorems 7.1 and 8.5], there are integers a_i, $1 \le i \le d-1$, such that $a_1 \ge \cdots \ge a_{d-1}$ and $f_*(\mathcal{O}_Y)/\mathcal{O}_{\mathbf{P}(V)} \cong \mathcal{O}_{\mathbf{P}(V)}(a_1) \oplus \cdots \oplus \mathcal{O}_{\mathbf{P}(V)}(a_{d-1})$. The restriction of $f_*(\mathcal{O}_Y)/\mathcal{O}_{\mathbf{P}(V)}$ to any line shows that the $(d-1)$-ple $(a_1, \dots, a_{d-1})$ is uniquely determined by f. Following the terminology used in the theory of Riemann surfaces we say that $(a_1, \dots, a_{d-1})$ are the scrollar invariants of f. We have $H^0(Y, \mathcal{O}_Y) = \mathbf{C}$ if and only if $a_1 < 0$. For every integer $t < -a_1$ the natural map $f^* : H^0(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(t)) \to H^0(Y, f^*(\mathcal{O}_{\mathbf{P}(V)}(t)))$ is an isomorphism. The natural map

$$f^* : H^0(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(-a_1)) \to H^0(Y, f^*(\mathcal{O}_{\mathbf{P}(V)}(-a_1)))$$

is not surjective and this is a characterization of the integer a_1.

Remark 2. Let $f : Y \to M$ be a c-flat finite covering of degree d. Let F be a locally free $\mathcal{O}_Y$-sheaf on Y with finite rank r and $P \in M$. For each $Q \in f^{-1}(P)$ there is a neighborhood U_Q of Q such that $F|U_Q \cong U_Q^{\oplus r}$. Since $f^{-1}(P)$ is finite, f is proper and Y is Hausdorff, we can find such neighborhoods U_Q with $U_Q \cap U_{Q'} = \emptyset$ for all $Q, Q' \in f^{-1}(P)$ with $Q \neq Q'$ and a neighborhood U of P such that $f^{-1}(U)$ is the disjoint union of such open sets U_Q, $Q \in f^{-1}(P)$. Since $f_*(\mathcal{O}_Y)$ is locally free of rank d, we see that $f_*(F)$ is locally free of rank rd. Hence if V is a Banach space such that $\mathbf{P}(V)$ is localizing and $M = \mathbf{P}(V)$, then $f_*(F) \cong \bigoplus_{i=1}^{rd} \mathcal{O}_{\mathbf{P}(V)}(t_i))$ for some integers t_i [7, Theorems 7.1 and 8.5]. Hence as in the previous example F is uniquely determined by rd integers.

Example 2. Let $f : Y \to M$ be a proper holomorphic map with finite fibers between finite-dimensional complex spaces. Assume M smooth. The map f is c-flat if and only Y is locally Cohen–Macaulay, i.e., if and only if for every $P \in Y$ the Noetherian local ring $\mathcal{O}_{Y,P}$ is Cohen–Macaulay [3, Example III.10.9]. If Y is smooth, then it is locally Cohen–Macaulay [3, p. 184]. Then we can obtain in an easy way several infinite-dimensional examples taking an infinite-dimensional complex space Z and taking $M' := M \times Z$, $Y' := Y \times Z$ and $f' : Y' \to M'$ induced by f.

Remark 3. (a) The composition of two c-flat maps is c-flat.

(b) A locally biholomorphic and proper map between two complex manifolds is c-flat.

Remark 4. Let $f : Y \to M$ be a proper map with finite fibers and F any sheaf on Y. By [5, Ch. 1, Theorem 4], we have $Rf^i_*(F) = 0$ for every $i > 0$.

Corollary 2. *Fix an integer $t \ge 1$. Let $f : Y \to M$ be a c-flat map between complex spaces. Assume that for every finite rank holomorphic vector bundle E on M we have $H^t(M, E) = 0$. Then $H^t(Y, F) = 0$ for every finite rank holomorphic vector bundle F on Y.*

Proof. By Remark 4 we have $Rf^i_*(F) = 0$ for every $i > 0$. Thus $H^t(Y, F) \cong H^t(M, f_*(F))$ by the Leray spectral sequence of f. By Remark 2 $f_*(F)$ is a locally free sheaf on M with finite rank. Hence $H^t(M, f_*(F)) = 0$ by our assumption on M. $\square$

Example 3. Fix an integer $d \geq 2$, a complex space M, a holomorphic line bundle L on M and an effective Cartier divisor D on M such that $\mathcal{O}_M(-D) \cong L^{\otimes d}$; we require that L be locally holomorphically trivial, i.e. our definition of line bundle be more restrictive than the one in [7, p. 490]. By [1, Example 1.1], these data are sufficient to construct a complex space Y and a proper holomorphic map $f : Y \to M$ such that $f_*(\mathcal{O}_Y) \cong \mathcal{O}_M \oplus L \oplus \cdots \oplus L^{\otimes(d-1)}$. Hence $f_*(\mathcal{O}_Y)$ is locally free. Thus f is c-flat if M is smooth. This is the classical construction of a cyclic branched covering with D as a ramification divisor. One can see Y as a closed analytic subset of the total space of the line bundle L^* and f is the restriction of the natural map $L^* \to M$ to Y. As in [1, Example 1.1], one can even give local equations for Y inside L^* in terms of local equations of D inside M. In particular, Y is isomorphic to an effective Cartier divisor of L^*. From these local equations it follows that if M and D are smooth, then Y are smooth. Fix an integer $t > 0$. By Remark 4 and the Leray spectral sequence of f we obtain $H^t(Y, \mathcal{O}_Y) \cong \bigoplus_{i=0}^{d-1} H^t(M, L^{\otimes i})$. Hence $H^t(Y, \mathcal{O}_Y) = 0$ if and only if $H^t(M, L^{\otimes i}) = 0$ for every integer i with $0 \leq i \leq d-1$.

The vanishing theorem for Dolbeaut cohomology groups corresponding to the next result is proved in [7, Theorem 7.3].

Proposition 1. *Let V be a complex Banach space. Then the sheaf cohomology group $H^1(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(x))$ vanishes for every $x \leq 0$.*

Proof. By [7, Theorem 7.1], for every holomorphic line bundle L on $\mathbf{P}(V)$ there is an integer t such that $L \cong \mathcal{O}_{\mathbf{P}(V)}(t)$. The sheaf cohomology group $H^1(Y, \mathcal{O}_Y^*)$ classifies the isomorphism classes of all line bundles on any reduced complex space Y. Thus $H^1(\mathbf{P}(V), \mathcal{O}^*_{\mathbf{P}(V)}) \cong \mathbf{Z}$.

Claim: We have $H^1(\mathbf{P}(V), \mathbf{Z}) = 0$.

Proof of the Claim: It would be easy to check that the first singular cohomology group vanishes just by checking that $\mathbf{P}(V)$ is simply-connected to obtain $H_0(\mathbf{P}(V), \mathbf{Z}) = \mathbf{Z}$, $H_1(\mathbf{P}(V), \mathbf{Z}) = 0$ and then by applying the universal-coefficient theorem [12, p. 243]. But for sheaf cohomology we follow a longer path. Since V and $V\backslash\{0\}$ are paracompact, sheaf cohomology and sheaf Čech cohomology on V and $V\backslash\{0\}$ are the same [4, p. 228]. By [12, p. 334], sheaf Čech cohomology and Alexander cohomology agree on V and $V\backslash\{0\}$. By [11, Corollary 1.3], the inclusion $V\backslash\{0\} \to V$ is a homotopy equivalence. V is contractible. Alexander cohomology satisfies the homotopy axiom [12, p. 314]. Hence $H^1(V\backslash\{0\}, \mathbf{Z}) = 0$ (sheaf cohomology). Let $\pi : V\backslash\{0\} \to \mathbf{P}(V)$ be the quotient map. Thus π is a locally trivial fibration with $\mathbf{C}^*$ as fiber. The spectral sequence of π induces an exact sequence for sheaf cohomology

$$0 \to H^1(\mathbf{P}(V), \pi_*(\mathbf{Z})) \to H^1(V\backslash\{0\}, \mathbf{Z}) \to H^0(\mathbf{P}(V), \pi_*(\mathbf{Z})) \tag{4}$$

(a part of the five term exact sequence arising from any first quadrant spectral sequence). Thus $H^1(\mathbf{P}(V), \pi_*(\mathbf{Z})) = 0$. Hence to check the Claim it is sufficient to prove that $\pi_*(\mathbf{Z}) \cong \mathbf{Z}$. Notice that $\pi_*(\mathbf{Z})$ is locally isomorphic to the constant sheaf $\mathbf{Z}$. To check that it is globally isomorphic to $\mathbf{Z}$ one can either use the simply-connectedness of $\mathbf{P}(V)$ (to prove it use the simply-connectedness of $V\backslash\{0\}$ just proven and the long exact sequence of homotopy groups associated to the fibration π [13]) or the fact that it has a global nowhere vanishing section because $H^0(\mathbf{P}(V), \pi_*(\mathbf{Z})) = H^0(V\backslash\{0\}, \mathbf{Z}) = \mathbf{Z}$. Hence the proof of the Claim is finished.

From the Claim and the exponential sequence (2) we obtain

$$H^1(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}) = 0.$$

Now assume $x < 0$. There is a reduced hypersurface T of $\mathbf{P}(V)$ with $\deg(T) = -x$. Hence $H^0(T, \mathcal{O}_T) = \mathbf{C}$. Thus the restriction map $H^0(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}) \to H^0(T, \mathcal{O}_T)$ is surjective. From the exact sequence

$$0 \to \mathcal{O}_{\mathbf{P}(V)}(x) \to \mathcal{O}_{\mathbf{P}(V)} \to \mathcal{O}_T \to 0 \tag{5}$$

we obtain $H^1(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(x)) = 0$. $\square$

The referee made the following very useful remark.

Remark 5 (due to the referee). Let V be an infinite-dimensional Banach space such that $\mathbf{P}(V)$ admits smooth partitions of unity. Then for every integer x the Čech cohomology group $\check{H}^1(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(x))$ is naturally embedded into the Dolbeaut group $H^{0,1}(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(x))$ which vanishes by [6, Theorem 7.3]. Hence if $\mathbf{P}(V)$ admits smooth partitions of unity, then Proposition 1 is true also for every positive integer x. V admits smooth partitions of unity if it is a separable Hilbert space; for more examples see [7, p. 511], or [2, §8], and the references therein.

Remark 6. Let V be an infinite-dimensional complex Banach space. Fix integers d, y such that $d \geq 2$ and $y < 0$. Let D be a closed and reduced hypersurface of $\mathbf{P}(V)$ with $\deg(D) = -yd$. Make the construction of Example 3 taking $Y = \mathbf{P}(V)$ and $L = \mathcal{O}_{\mathbf{P}(V)}(y)$. By Proposition 1 we have $H^1(\mathbf{P}(V), \mathcal{O}_{\mathbf{P}(V)}(iy)) = 0$ for every integer i such that $0 \leq i \leq d-1$. Hence $H^1(Y, \mathcal{O}_Y) = 0$. Thus by the exponential sequence (2) every topologically trivial holomorphic line bundle on Y is holomorphically trivial. Now fix an integer $x < 0$ and set $R := f^*(\mathcal{O}_{\mathbf{P}(V)}(x)) \in \mathrm{Pic}(Y)$. For every locally free sheaf A on $\mathbf{P}(V)$ we have $f_*(f^*(A)) \cong f_*(\mathcal{O}_Y) \otimes A$ (projection formula). Thus $f_*(R) \cong \bigoplus_{i=0}^{d-1} \mathcal{O}_{\mathbf{P}(V)}(x + iy)$. Hence by Proposition 2, Remark 4 and the Leray spectral sequence of f we obtain $H^1(Y, R) = 0$.

Acknowledgement

I want to thank the referee for several very useful remarks.

The author was partially supported by MURST and GNSAGA of INdAM (Italy).

References

1. F. Catanese and C. Ciliberto, On the irregularity of cyclic coverings of algebraic surfaces. *Geometry of complex projective varieties* (*Cetraro,* 1990), 89–115, *Sem. Conf.,* 9, *Mediterranean, Rende,* 1993.
2. R. Fry and S. McManus, Smooth bump functions and the geometry of Banach spaces: a brief survey. *Expo. Math.* **20**(2002), No. 2, 143–183.
3. R. Hartshorne, Algebraic geometry. *Graduate Texts in Mathematics,* No. 52. *Springer-Verlag, New York–Heidelberg,* 1977.
4. R. Godement, Théorie des faisceaux. *Hermann, Paris,* 1973.
5. H. Grauert and R. Remmert, Theorie der Steinschen Räume. *Grundlehren der Mathematischen Wissenschaften,* 227. *Springer-Verlag, Berlin–New York,* 1977.
6. B. Kotzev, Vanishing of the first cohomology group of line bundles on complete intersections in infinite-dimensional projective space. *Ph. D. thesis, University of Purdue,* 2001.
7. L. Lempert, The Dolbeaut complex in infinite dimension, I. *J. Amer. Math. Soc.* **11**(1998), No. 3, 485–520.
8. L. Lempert, The Dolbeaut complex in infinite dimension. III. Sheaf cohomology in Banach spaces. *Invent. Math.* **142**(2000), No. 3, 579–603.
9. P. Mazet, Analytic sets in locally convex spaces. *North-Holland, Amsterdam,* 1984.
10. J.-P. Ramis, Sous-ensembles analytiques d'une variété analytique Banachique. (French) *Séminaire Pierre Lelong (Analyse) (année* 1967–1968), 140–164. *Lecture Notes in Math.,* Vol. 71, *Springer, Berlin,* 1968.
11. G. Ruget, A propos des cycles analytiques de dimension infinie. *Invent. Math.* **8**(1969), 267–312.
12. E. H. Spanier, Algebraic topology. *McGraw-Hill Book Co., New York–Toronto, Ont.–London,* 1966.
13. G. W. Whitehead, Elements of homotopy theory. *Graduate Texts in Mathematics,* 61. *Springer-Verlag, New York–Berlin,* 1978.

(Received 14.01.2002; revised 24.07.2002)

Author's address:

Dept. of Mathematics
University of Trento
38050 Povo (TN)
Italy
E-mail: ballico@science.unitn.it