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ON THE SOLVABILITY OF A SPATIAL PROBLEM OF DARBOUX TYPE FOR THE WAVE EQUATION

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ABSTRACT. The question of the correct formulation of one spatial problem of Darboux type for the wave equation has been investigated. The correct formulation of that problem in the Sobolev space has been proved for surfaces having a quite definite orientation on which are given the boundary value conditions of the problem of Darboux type.

In the space of variables x_1, x_2, t we consider the wave equation

$$\square u \equiv \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x_1^2} - \frac{\partial^2 u}{\partial x_2^2} = F, \tag{1}$$

where F is the known and u is the unknown function.

Denote by $D_+ : 0 < x_2 < t$, $0 < t < t_0$, the domain lying in a half-space $t > 0$ bounded by a time-type plane surface $S_0 : x_2 = 0$, $0 \le t \le t_0$, a characteristic surface $S_1 : t - x_2 = 0$, $0 \le t \le t_0$, of equation (1), and a plane $t = t_0$.

Consider the problem of Darboux type formulated as follows: find in the domain D_+ the solution $u(x_1, x_2, t)$ of equation (1) by the following boundary conditions:

$$u\big|_{S_1} = f_1 \tag{2}$$

and

$$\frac{\partial u}{\partial n}\Big|_{S_0} = 0, \tag{3}$$

where f_1 is a given real function and $\frac{\partial}{\partial n}$ is the derivative with respect to the outer normal to S_0.

Note that in the case where S_0 is either a characteristic surface $S_2 :$ $t + x_2 = 0$, $0 \le t \le t_0$ or a plane surface $S_2 : kt + x_2 = 0$, $0 \le t \le t_0$,

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$|k| < 1$ of timetype, the problem (1)-(3) in which the boundary condition (3) is replaced by the condition

$$u\big|_{S_2} = f_2 \tag{4}$$

is studied in [1-4]. Other multidimensional analogues of the Darboux problem are considered in [5-7].

Denote by $C_*^\infty(\overline{D}_+)$ the space of functions in the class $C^\infty(\overline{D}_+)$ having bounded supports, i.e.,

$$C_*^\infty(\overline{D}_+) = \big\{u \in C^\infty(\overline{D}_+) \,:\, \operatorname{diam}\operatorname{supp} u < +\infty\big\}.$$

The spaces $C_*^\infty(S_i)$, $i = 0, 1, 2$, are defined in a similar manner.

It is known that the spaces $C_*^\infty(\overline{D}_+)$, $C_*^\infty(S_i)$, $i = 0, 1, 2$, are dense everywhere in the Sobolev spaces $W_2^k(D_+)$, $W_2^k(S_i)$, $i = 0, 1, 2$, where $k \geq 0$ is integer [8].

Lemma 1. *For any $u \in W_2^2(D_+)$, satisfying the homogeneous boundary condition* (3)*, the a priori estimate*

$$\|u\|_{W_2^1(D_+)} \leq C\big(\|f_1\|_{W_2^1(S_1)} + \|F\|_{L_2(D_+)}\big) \tag{5}$$

is valid, where $f_1 = u|_{S_1}$, $F = \Box u$, and C is a positive constant not depending on u.

Proof. Denote by $D_- \,:\, -t < x_2 < 0$, $0 < t < t_0$, the domain symmetric to D_+ with respect to the plane $x_2 = 0$ and by $D \,:\, -t < x_2 < t$, $0 < t < t_0$, the domain which is the union of the domains D_+ and D_- with a part of a plane surface $x_2 = 0$, $0 < t < t_0$.

It is easy to verify that if one continue evenly the function satisfying the boundary condition (3), then the function u_0 obtained in D

$$u_0(x_1, x_2, t) = \begin{cases} u(x_1, x_2, t), & x_2 \geq 0, \\ u(x_1, -x_2, t), & x_2 < 0 \end{cases} \tag{6}$$

will belong to the class $W_2^2(D)$. According to the results in [3], the function $u_0 \in W_2^2(D)$ satisfies the following a priori estimate:

$$\|u_0\|_{W_2^1(D)} \leq C\big(\|f_1\|_{W_2^1(S_1)} + \|f_2\|_{W_2^1(S_2)} + \|F_0\|_{L_2(D)}\big), \tag{7}$$

where $f_i = u_0|_{S_i}$, $i = 1, 2$, $F_0 = \Box u_0$, and $S_2 \,:\, t + x_2 = 0$, $0 \leq t \leq t_0$, is a part of a boundary of D, appearing in the boundary condition (4).

It remains only to note that in virtue of (6) in the estimate (7)

$$\|u_0\|_{W_2^1(D)} = 2\|u\|_{W_2^1(D_+)}, \quad \|f_2\|_{W_2^1(S_2)} = \|f_1\|_{W_2^1(S_1)},$$
$$\|F_0\|_{L_2(D)} = 2\|F\|_{L_2(D_+)}. \quad \Box$$

Below we shall prove the following

Lemma 2. *For any* $f_1 \in C_*^\infty(S_1)$ *and* $F \in C_*^\infty(\overline{D}_+)$ *satisfying the conditions*

$$\frac{\partial^k F}{\partial n^k}\Big|_{S_0} = 0, \quad k = 1, 3, 5, \dots, \tag{8}$$

the problem (1)–(3) *can be solved uniquely in the class* $C_*^\infty(\overline{D}_+)$.

If one continues evenly the function $F \in C_*^\infty(\overline{D}_+)$ in D_-, then in virtue of (8) the function F_0 obtained in D

$$F_0(x_1, x_2, t) = \begin{cases} F(x_1, x_2, t), & x_2 \geq 0, \\ F(x_1, -x_2, t), & x_2 < 0 \end{cases}$$

will belong to the class $C_*^\infty(\overline{D})$. Denote by f_2 the function defined on $S_2 : t + x_2 = 0$, $0 \leq t \leq t_0$, by the equality

$$f_2\big|_{S_2} = f_2(x_1, x_2, -x_2) = f_1(x_1, -x_2, -x_2) = f_1\big|_{S_1}. \tag{9}$$

Consider now in D the problem of determining the solution $u_0(x_1, x_2, t)$ of the equation

$$\Box u_0 \equiv \frac{\partial^2 u_0}{\partial t^2} - \frac{\partial^2 u_0}{\partial x_1^2} - \frac{\partial^2 u_0}{\partial x_2^2} = F_0 \tag{10}$$

belonging to the class $C_*^\infty(\overline{D})$ by the boundary conditions

$$u_0\big|_{S_i} = f_i, \quad i = 1, 2. \tag{11}$$

Note that the integral representation for regular solutions of the problem (10),(11) is obtained in [3]. On the basis of this representation the conclusion on the solvability of the problem in the class $C_*^\infty(\overline{D})$ is made without proof. To prove the conclusion completely, below we shall reduce the spatial problem (10),(11) to the plane Goursat problem with a parameter. For the solution of the problem, necessary estimates depending on the parameter will be obtained.

If u_0 is a solution of the problem (10),(11) of the class $C_*^\infty(\overline{D})$, then after the Fourier transform with respect to the variable x_1, equation (10) and the boundary conditions (11) take the form

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x_2^2} + \lambda^2 v = \Phi, \tag{12}$$

$$v\big|_{\ell_i} = g_i, \quad i = 1, 2, \tag{13}$$

where

$$v(\lambda, x_2, t) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{\infty} u_0(x_1, x_2, t) e^{-ix_1\lambda}\, dx_1$$

is the Fourier transform of the function $u_0(x_1,x_2,t)$ and Φ, g_1, g_2 is the Fourier transform of the functions F_0, f_1, f_2 with respect to x_1. Here ℓ_1 : $t-x_2=0$, $0\le t\le t_0$, ℓ_2 : $t+x_2=0$, $0\le t\le t_0$, are segments of rays lying in the plane of variables x_2,t and coming from the origin $O(0,0)$.

Thus after the Fourier transform with respect to x_1 the spatial problem (10),(11) is reduced to the plane Goursat problem (12),(13) with a parameter λ in the domain D_0 : $-t<x_2<t$, $0<t<t_0$, of the plane of variables x_2,t.

Remark 1. If $u_0(x_1,x_2,t)$ is the solution of problem (10),(11) of the class $C_*^\infty(\overline{D})$, then $v(\lambda,x_2,t)$ will be the solution of the problem (12),(13) of the class $C^\infty(\overline{D}_0)$ which at the same time according to Paley–Wiener theorem is an entire analytic function with respect to λ satisfying the following growth condition: for an integer $N\ge 0$ there is a constant K_N such that [8,9]

$$|v(\lambda,x_2^0,t^0)|\le K_N(1+|\lambda|^2)^{-N}e^{d|\operatorname{Im}\lambda|}, \tag{14}$$

where

$$d=d(x_2^0,t^0)=\max_{(x_1,x_2^0,t^0)\in\operatorname{supp}u_0}|x_1|;$$

moreover, as the constant K_N one can take the value [9]

$$K_N=K_N(x_2^0,t^0)=\frac{1}{\sqrt{2\pi}}\int\limits_{|x_1|<d}\Big|\Big(1-\frac{\partial^2}{\partial x_1^2}\Big)^N u_0(x_1,x_2^0,t^0)\Big|\,dx_1.$$

According to the same theorem, if $v(\lambda,x_2,t)$ belongs to the class $C^\infty(\overline{D}_0)$ with respect to the variables x_2,t for fixed λ, while with respect to λ it is an entire analytic function satisfying the estimate (14) for some $d=const>0$, then the function $u_0(x_1,x_2,t)$, being the inverse Fourier transform of the function $v(\lambda,x_2,t)$, belongs to the class $C_*^\infty(\overline{D})$.

According to our assumptions, estimates analogous to (14) are valid for the functions Φ, g_1, g_2 which belong respectively to the classes $C^\infty(\overline{D}_0)$, $C^\infty(\ell_1)$, $C^\infty(\ell_2)$ and are entire analytic functions with respect to λ.

In the new variables

$$\xi=\frac12(t+x_2),\quad \eta=\frac12(t-x_2), \tag{15}$$

retaining the same notations for the functions v,Φ,g_i, the problem (12),(13) takes the form

$$\frac{\partial^2 v}{\partial\xi\partial\eta}+\lambda^2 v=\Phi, \tag{16}$$

$$v\big|_{\gamma_i}=g_i,\quad i=1,2. \tag{17}$$

Here the solution $v = v(\lambda, \xi, \eta)$ of equation (16) is considered in the domain Ω_0 of the plane of variables ξ, η being the image of the domain D_0 for linear transform (15), and γ_i being the image of ℓ_i for the same transform. Obviously, the domain Ω_0 is a triangle OP_1P_2 witha vertices $O(0,0)$, $P_1(t_0, 0)$, $P_2(0, t_0)$ and

$$\gamma_1 \ : \ \eta = 0, \quad 0 \leq \xi \leq t_0, \qquad \gamma_2 \ : \ \xi = 0, \quad 0 \leq \eta \leq t_0$$

are the sides of OP_1, OP_2.

As is well known, under the assumptions with respect to the functions Φ, g_i the problem (16),(17) has a unique solution of the class $C^\infty(\overline{\Omega_0})$ which can be represented in the form [10]

$$\begin{aligned} v(\lambda, \xi, \eta) = {} & R(\xi, 0; \xi, \eta) g_1(\lambda, \xi) + R(0, \eta; \xi, \eta) g_2(\lambda, \eta) - \\ & - R(0,0;\xi,\eta) g_1(\lambda, 0) - \int\limits_0^\xi \frac{\partial R(\sigma, 0; \xi, \eta)}{\partial \sigma} \, g_1(\lambda, \sigma) \, d\sigma - \\ & - \int\limits_0^\eta \frac{\partial R(0, \tau; \xi, \eta)}{\partial \tau} \, g_2(\lambda, \tau) \, d\tau + \\ & + \int\limits_0^\xi d\sigma \int\limits_0^\eta R(\sigma, \tau; \xi, \eta) \Phi(\lambda, \sigma, \tau) \, d\tau, \end{aligned} \tag{18}$$

where $g_1(\lambda, \xi) = v(\lambda, \xi, 0)$, $0 \leq \xi \leq t_0$, $g_2(\lambda, \eta) = v(\lambda, 0, \eta)$, $0 \leq \eta \leq t_0$ are the Goursat data for v and $R(\xi_1, \eta_1; \xi, \eta)$ are the Riemann functions for equation (16).

The Riemann function $R(\xi_1, \eta_1; \xi, \eta)$ for equation (16), as is known, can be expressed in terms of the Bessel function $\mathcal{I}_0$ of zero order as [11]

$$R(\xi_1, \eta_1; \xi, \eta) = \mathcal{I}_0\big(2\lambda\sqrt{(\xi - \xi_1)(\eta - \eta_1)}\,\big). \tag{19}$$

Remark 2. Since the Bessel function $\mathcal{I}_0(z)$ of the complex argument z is an entire analytic function, the formula (18) in virtue of the equality (19) gives the solution of equation (16) satisfying the Goursat data

$$\begin{aligned} v(\lambda, \xi, 0) &= g_1(\xi), \quad 0 \leq \xi \leq t_0, \\ v(\lambda, 0, \eta) &= g_2(\eta), \quad 0 \leq \eta \leq t_0. \end{aligned} \tag{20}$$

The solution is an entire analytic function with respect to the complex parameter λ.

From the known representation of the Bessel function [12]

$$\mathcal{I}_0(z) = \frac{1}{2\pi} \int\limits_{-\pi}^{\pi} \exp(iz\sin\theta)\, d\theta \tag{21}$$

we can easily get that

$$\mathcal{I}_0'(z) = -\frac{z}{2\pi} \int\limits_{-\pi}^{\pi} \cos^2\theta \exp(iz\sin\theta)\, d\theta,$$

whence

$$\frac{d\mathcal{I}_0(2\lambda\sqrt{\nu x}\,)}{dx} = -\frac{\lambda^2\nu}{\pi} \int\limits_{-\pi}^{\pi} \cos^2\theta \exp(i2\lambda\sqrt{\nu x}\,\sin\theta)\, d\theta. \tag{22}$$

Now, from (19),(21) and (22) we immediately get the following equalities and estimates

$$\begin{gathered}
R(\xi,0;\xi,\eta) = R(0,\eta;\xi,\eta) = 1, \\
R(0,0;\xi,\eta) \le \exp(2\sqrt{\xi\eta}\,|\operatorname{Im}\lambda|) \le \exp(2t_0|\operatorname{Im}\lambda|), \\
\Big|\frac{\partial R(\sigma,0;\xi,\eta)}{\partial\sigma}\Big| \le 2|\lambda|^2\eta\exp(2\sqrt{\xi\eta}\,|\operatorname{Im}\lambda|) \le 2|\lambda|^2t_0\exp(2t_0|\operatorname{Im}\lambda|), \\
\Big|\frac{\partial R(0,\tau;\xi,\eta)}{\partial\tau}\Big| \le 2|\lambda|^2\xi\exp(2\sqrt{\xi\eta}\,|\operatorname{Im}\lambda|) \le 2|\lambda|^2t_0\exp(2t_0|\operatorname{Im}\lambda|), \\
|R(\sigma,\tau;\xi,\eta)| \le \exp(2\sqrt{\xi\eta}\,|\operatorname{Im}\lambda|) \le \exp(2t_0|\operatorname{Im}\lambda|).
\end{gathered}$$

From this, without loss of generality and assuming the the estimate (14) with respect to λ and the same constants K_N and d are valid in virtue of our assumptions for the functions Φ, g_1, g_2, for the solution $v(\lambda,\xi,\eta)$ of the problem (16),(17) representable in the form of (18)m we obtain the following estimates:

$$\begin{aligned}
|v(\lambda,\xi,\eta)| &\le |g_1(\lambda,\xi)| + |g_2(\lambda,\eta)| + |g_1(\lambda,0)|\exp(2t_0|\operatorname{Im}\lambda|) + \\
&+ 2|\lambda|^2t_0\exp(2t_0|\operatorname{Im}\lambda|)\int\limits_0^{\xi}|g_1(\lambda,\sigma)|d\sigma + \\
&+ 2|\lambda|^2t_0\exp(2t_0|\operatorname{Im}\lambda|)\int\limits_0^{\eta}|g_2(\lambda,\tau)|d\tau + \\
&+ \exp(2t_0|\operatorname{Im}\lambda|)\int\limits_0^{\xi}d\sigma\int\limits_0^{\eta}|\Phi(\lambda,\sigma,\tau)|d\tau \le
\end{aligned}$$

$$
\begin{aligned}
&\le 2K_N(1+|\lambda|^2)^{-N}\exp(d|\operatorname{Im}\lambda|)+\\
&+\exp(2t_0|\operatorname{Im}\lambda|)K_N(1+|\lambda|^2)^{-N}\exp(d|\operatorname{Im}\lambda|)+\\
&+2|\lambda|^2t_0\exp(2t_0|\operatorname{Im}\lambda|)\xi K_N(1+|\lambda|^2)^{-N}\exp(d|\operatorname{Im}\lambda|)+\\
&+2|\lambda|^2t_0\exp(2t_0|\operatorname{Im}\lambda|)\eta K_N(1+|\lambda|^2)^{-N}\exp(d|\operatorname{Im}\lambda|)+\\
&+\exp(2t_0|\operatorname{Im}\lambda|)\xi\eta K_N(1+|\lambda|^2)^{-N}\exp(d|\operatorname{Im}\lambda|)\le\\
&\le \widetilde{K}_{N-1}(1+|\lambda|^2)^{N-1}\exp(\widetilde{d}|\operatorname{Im}\lambda|). \qquad (23)
\end{aligned}
$$

Here

$$
\begin{gathered}
\widetilde{K}_{N-1}=(3+5t_0^2)K_N,\quad \widetilde{d}=2t_0+d,\\
d=\max_{(x_1,x_2,t)\in I}|x_1|,\quad I=\operatorname{supp}F_0\cup\operatorname{supp}f_1\cup\operatorname{supp}f_2,\\
K_N=\frac{1}{2\pi}\int\limits_{|x_1|<d}\max_{0\le i\le 2}\max_{(x_2^0,t^0)\in D_0}|\varphi_i(x_1,x_2^0,t^0)|\,dx_1,\\
\varphi_0=\Big(1-\frac{\partial^2}{\partial x_1^2}\Big)^N F_0,\quad \varphi_i=\Big(1-\frac{\partial^2}{\partial x_1^2}\Big)^N f_i,\ \ i=1,2.
\end{gathered}
$$

In virtue of the estimate (23), the function $v(\lambda,\xi,\eta)$ according to the Paley–Wiener theorem, turning to the original variables x_2,t, by the formula (15) will be the Fourier transform of a function $u_0(x_1,x_2,t)$ of the class $C_*^\infty(\overline{D})$; moreover, in virtue of (12),(13) the function $u_0(x_1,x_2,t)\in C_*^\infty(\overline{D})$ will be the solution of the problem (10),(11). Let us show now that the restriction of that function to the domain D_+, i.e., $u=u_0|_{D_+}$, is the solution of the problem (1)–(3) of the class $C_*^\infty(\overline{D}_+)$. To this end let us prove that the function $u_0(x_1,x_2,t)$ is even with respect to x_2. Because the function F_0 is even with respect to x_2 and the functions f_1 and f_2 are connected by the equality (9), we can easily verify that the function $\widetilde{u}(x_1,x_2,t)=u_0(x_1,-x_2,t)$ is also the solution of the problem (10),(11) of the class $C_*^\infty(\overline{D})$. But in virtue of a priori estimate (7) the problem (10),(11) cannot have more than one solution of the above-mentioned class. Therefore $\widetilde{u}(x_1,x_2,t)\equiv u_0(x_1,x_2,t)$, i.e., the solution $u_0(x_1,x_2,t)$ of equation (10) is an even function with respect to x_2. From this it immediately follows that $\frac{\partial u_0}{\partial n}\big|_{x_2=0}=0$, i.e., the boundary condition (3) is fulfilled for $u=u_0|_{D_+}$. Thus the function $u=u_0|_{D_+}\in C_*^\infty(\overline{D}_+)$ is the solution of the problem (1)-(3). The uniqueness of the solution follows from a priori estimate (5). □

Definition. Let $f_1\in W_2^1(S_1)$, $F\in L_2(D_+)$. The function $u\in W_2^1(D)$ will be called a strong solution of the problem (1)–(3) of the class W_2^1, if there exists a sequence $u_n\in C_*^\infty(\overline{D}_+)$ such that $\frac{\partial u_n}{\partial n}\big|_{S_0}=0$, $u_n\to u$ in the space $W_2^1(D_+)$ and $\Box u \to F$ in the space $L_2(D_+)$, i.e., for $n \to \infty$

$$\|u_n - u\|_{W_2^1(D_+)} \to 0, \quad \|\Box u_n - F\|_{L_2(D_+)} \to 0,$$
$$\|u_n|_{S_1} - f_1\|_{W_2^1(S_1)} \to 0.$$

We have the following

Theorem 1. *For any $f_1 \in W_2^1(S_1)$, $F \in L_2(D_+)$ there exists a unique strong solution u of the problem (1)–(3) of the class W_2^1 for which the estimate (5) is valid.*

Proof. It is known that the space $C_0^\infty(D_+) \subset C_*^\infty(\overline{D}_+)$ of infinitely differentiable finite functions in D_+ is everywhere dense in $L_2(D_+)$, while the space $C_*^\infty(S_1)$ is dense in $W_2^1(S_1)$. Therefore there exist sequences $F_n \in C_0^\infty(D_+)$ and $f_{1n} \in C_*^\infty(S_1)$ such that

$$\lim_{n\to\infty} \|F - F_n\|_{L_2(D_+)} = \lim_{n\to\infty} \|f_1 - f_{1n}\|_{W_2^1(S_1)} = 0. \tag{24}$$

Since the functions $F_n \in C_0^\infty(D_+)$ satisfy the conditions (8), according to Lemma 2 there exists a sequence $u_n \in C_*^\infty(\overline{D}_+)$ of solutions of the problem (1)–(3) for $F = F_n$, $f_1 = f_{1n}$.

In virtue of the inequality (5) we have

$$\|u_n - u_m\|_{W_2^1(D_+)} \le C\big(\|f_{1n} - f_{1m}\|_{W_2^1(S_1)} + \\ + \|F_n - F_m\|_{L_2(D_+)}\big). \tag{25}$$

It follows from (24) and (25) that the sequence of functions u_n is fundamental in the space $W_2^1(D_+)$. Therefore because the space $W_2^1(D_+)$ is complete, there exists a function $u \in W_2^1(D_+)$ such that $\frac{\partial u_n}{\partial n}\big|_{S_0} = 0$, $u_n \to u$ in $W_2^1(D_+)$, $\Box u_n \to F$ in $L_2(D_+)$ and $u_n|_{S_1} \to f_1$ in $W_2^1(S_1)$ for $n \to \infty$. Hence the function u is a strong solution of the problem (1)–(3) of the class W_2^1. The uniqueness of the strong solution of the problem (1)–(3) of the class W_2^1 follows from the inequality (5). $\Box$

Consider now the case where in equation (1) we have the lowest terms

$$Lu \equiv \Box u + au_{x_1} + bu_{x_2} + cu_t + du = F, \tag{26}$$

where the coefficients a, b, c, and d are the given bounded measurable functions in the domain D_+.

In the space $W_2^1(D_+)$ let us introduce an equivalent norm

$$\|u\|^2_{D_+,1,\gamma} = \int\limits_{D_+} e^{-\gamma t}(u + u_t^2 + u_{x_1}^2 + u_{x_2}^2)\, dx\, dt, \quad \gamma > 0,$$

depending on the parameter γ. The norms $\|F\|_{D_+,0,\gamma}$ and $\|f_1\|_{S_1,1,\gamma}$ in the spaces $L_2(D_+)$ and $W_2^1(S_1)$ are introduced analogously.

Arguments similar to those given in [4] allow us to prove the validity of the following

Lemma 3. *For any $u \in W_2^1(D_+)$ satisfying the boundary condition* (3), *the a priori estimate*

$$\|u\|_{D_+,1,\gamma} \leq \frac{C}{\sqrt{\gamma}}\big(\|f_1\|_{S_1,1,\gamma} + \|F\|_{D_+,0,\gamma}\big) \tag{27}$$

holds, where $f_1 = u|_{S_1}$, $F = \Box u$, and the positive constant C does not depend on u and the parameter γ.

In virtue of the estimate (27), the lowest term of the above-introduced equivalent norms of spaces $L_2(D_+)$, $W_2^1(D_+)$, $W_2^1(S_1)$ in equation (26) for sufficiently large parameter γ give arbitrarily small perturbations which on the basis of Theorem 1 and the results of [4] allow us to prove that the problems (26),(2),(3) are uniquely solvable in the class W_2^1.

The following theorem holds.

Theorem 2. *For any $f_1 \in W_2^1(S_1)$, $F \in L_2(S_+)$ there exists a unique strong solution u of the problem* (26), (2), (3) *of the class W_2^1 for which the estimate* (5) *is valid.*

References

1. J. Hadamard, Lectures on Cauchy's problem in linear partial differential equations. *Yale Univ. Press, New Haven; Oxford Univ. Press, London,* 1923.

2. J. Tolen, Probleme de Cauchy sur la deux hipersurfaces caracteristiques secantes. *C. R. Acad. Sci. Paris, Ser. A-B* **291**(1980), No. 1, 49–52.

3. S. S. Kharibegashvili, On a characteristic problem for the wave equation. *Proc. I. Vekua Inst. Appl. Math. Tbilisi St. Univ.* **47**(1992), 76–82.

4. S. Kharibegashvili, On a spatial problem of Darboux type for second order hyperbolic equation. *Georgian Math. J.* **2**(1995), No. 3, 299–311.

5. A. V. Bitsadze, On mixed type equations on three-dimensional domains. (Russian) *Dokl. Akad. Nauk SSSR* **143**(1962), No. 5, 1017–1019.

6. A. M. Nakhushev, A multidimensional analogy of the Darboux problem for hyperbolic equations. (Russian) *Dokl. Akad. Nauk SSSR* **194**(1970), No. 1, 31–34.

7. T. Sh. Kalmenov, On multidimensional regular boundary value problems for the wave equation. (Russian) *Izv. Akad. Nauk Kazakh. SSR, Ser. Fiz.-Mat.* (1982), No. 3, 18–25.

8. L. Hörmander, Linear partial differential operators. *Grundl. Math. Wiss. Band* 116, *Springer-Verlag, Berlin–Heidelberg–New York,* 1963.

9. V. S. Vladimirov, Generalized functions in mathematical physics. (Russian) *Nauka, Moscow,* 1976.

10. A. V. Bitsadze, Some classes of partial differential equations. (Russian) *Nauka, Moscow,* 1981.

11. R. Courant, Partial differential equations (Methods of mathematical physics, ed. R. Courabt and D. Hilbert, Vol. 2), Interscience, *New York-London,* 1962.

12. F. Olver, Introduction to asymptotics and special functions. *Academic Press, New York-London,* 1974.

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