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ON SOME MULTIDIMENSIONAL VERSIONS OF A CHARACTERISTIC PROBLEM FOR SECOND-ORDER DEGENERATING HYPERBOLIC EQUATIONS

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Abstract. Some multidimensional versions of a characteristic problem for second-order degenerating hyperbolic equations are considered. Using the technique of functional spaces with a negative norm, the correctness of these problems in the Sobolev weighted spaces are proved.

In the space of variables x_1, x_2, t let us consider a second-order degenerating hyperbolic equation of the kind

$$Lu \equiv u_{tt} - t^m(u_{x_1x_1} + u_{x_2x_2}) + a_1u_{x_1} + a_2u_{x_2} + a_3u_t + a_4u = F, \quad (1)$$

where a_j, $j = 1, \dots, 4$, F are the given functions and u is the unknown real function, $m = \text{const} > 0$.

Denote by

$$D : 0 < t < \Big[1 - \frac{2+m}{2}r\Big]^{\frac{2}{2+m}}, \quad r = (x_1^2 + x_2^2)^{\frac{1}{2}} < \frac{2}{2+m}$$

a bounded domain lying in a half-space $t > 0$, bounded above by the characteristic conoid

$$S : t = \Big[1 - \frac{2+m}{2}r\Big]^{\frac{2}{2+m}}, \quad r \le \frac{2}{2+m}$$

of equation (1) with the vertex at the point $(0, 0, 1)$, and below by the base

$$S_0 : t = 0, \quad r \le \frac{2}{2+m}$$

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of that conoid; equation (1) has on S_0 a non-characteristic degeneration. In what follows, the coefficients a_i, $i=1,\dots,4$, of equation (1) in D are assumed to be the functions of the class $C^2(\overline{D})$.

For equation (1), consider a multidimensional version of the characteristic problem which is formulated as follows: On the domain D, find a solution $u(x_1,x_2,t)$ of equation (1) satisfying the boundary condition

$$u\big|_S = 0. \tag{2}$$

As will be shown below, the following Cauchy problem on finding in D a solution of equation

$$L^*v \equiv v_{tt} - t^m(v_{x_1x_1} + v_{x_2x_2}) - (a_1v)_{x_1} - (a_2v)_{x_2} - (a_3v)_t + a_4v = F \tag{3}$$

by the initial conditions

$$v\big|_{S_0} = 0, \quad v_t\big|_{S_0} = 0 \tag{4}$$

is the problem conjugate to problem (1), (2), where L^* is the operator formally conjugate to the operator L.

Note that for $m=0$, when equation (1) is non-degenerating and contains in its principal part a wave operator, some multidimensional Goursat and Darboux problems have been investigated in [1–6]. For a hyperbolic equation of second-order with non-characteristic degeneration of the kind

$$u_{tt} - |x_2|^m u_{x_1x_1} - u_{x_2x_2} + a_1u_{x_1} + a_2u_{x_2} + a_3u_t + a_4u = F,$$

as well as for a hyperbolic equation of second-order with characteristic degeneration

$$u_{tt} - u_{x_1x_1} - \big(|x_2|^m u_{x_2}\big)_{x_2} + a_1u_{x_1} + a_2u_{x_2} + a_3u_t + a_4u = F$$

the multidimensional variants of the Darboux problem are respectively studied in [7] and [8]. Other variants of multidimensional Goursat and Darboux problems can be found in [9–11].

Denote by E and E^* the classes of functions from the Sobolev space $W_2^2(D)$, satisfying respectively the boundary condition (2) or (4) and vanishing in some (own for every function) three-dimensional neighborhood of the circle $\Gamma = S\cap S_0 : r=\frac{2}{2+m}$, $t=0$ and of the segment $I: x_1=x_2=0$, $0\le t\le 1$. Let $W_+(W_+^*)$ be a Hilbert space with weight, obtained by closing the space $E(E^*)$ in the norm

$$\|u\|_1^2 = \int\limits_D \big[u_t^2 + t^m(u_{x_1}^2 + u_{x_2}^2) + u^2\big]dD.$$

Denote by $W_-(W_-^*)$ a space with negative norm which is constructed with respect to $L_2(D)$ and $W_+(W_+^*)$ [12].

Let $n = (\nu_1, \nu_2, \nu_0)$ be the unit vector of the outer to ∂D normal, i.e., $\nu_1 = \cos(\widehat{n, x_1})$, $\nu_2 = \cos(\widehat{n, x_2})$, $\nu_0 = \cos(\widehat{n, t})$. By definition, the derivative with respect to the conormal can be calculated on the boundary ∂D of the domain D for the operator L by the formula

$$\frac{\partial}{\partial N} = \nu_0 \frac{\partial}{\partial t} - t^m \nu_1 \frac{\partial}{\partial x_1} - t^m \nu_2 \frac{\partial}{\partial x_2}.$$

Remark 1. Since the derivative with respect to the conormal $\frac{\partial}{\partial N}$ for the operator L is an interior differential operator on the characteristic surfaces of equation (1), by virtue of (2) and (4) we have for the functions $u \in E$ and $v \in E^*$ that

$$\frac{\partial u}{\partial N}\Big|_S = 0, \quad \frac{\partial v}{\partial N}\Big|_{S_0} = 0. \tag{5}$$

Impose on the lower coefficients a_1 and a_2 in equation (1) the following restrictions:

$$M_i = \sup_{\overline{D}} \left| t^{-\frac{m}{2}} a_i(x_1, x_2, t) \right| < +\infty, \quad i = 1, 2. \tag{6}$$

Lemma 1. *For all functions $u \in E$, $v \in E^*$ the following inequalities hold:*

$$\|Lu\|_{W_-^*} \le c_1 \|u\|_{W_+}, \tag{7}$$

$$\|L^* v\|_{W_-} \le c_2 \|v\|_{W_+^*}, \tag{8}$$

where the positive constants c_1 and c_2 do not depend respectively on u and v, $\|\cdot\|_{W_+} = \|\cdot\|_{W_+^*} = \|\cdot\|_1$.

Proof. By the definition of a negative norm, for $u \in E$ with regard for equalities (2), (4) and (5) we have

$$\|Lu\|_{W_-^*} = \sup_{v \in W_+^*} \|v\|^{-1}_{W_+^*} (Lu, v)_{L_2(D)} = \sup_{v \in E^*} \|v\|^{-1}_{W_+^*} (Lu, v)_{L_2(D)} =$$

$$= \sup_{v \in E^*} \|v\|^{-1}_{W_+^*} \int\limits_D \big[u_{tt} v - t^m u_{x_1 x_1} v - t^m u_{x_2 x_2} v + a_1 u_{x_1} v + a_2 u_{x_1} v +$$

$$+ a_3 u_t v + a_4 u v\big] dD = \sup_{v \in E^*} \|v\|^{-1}_{W_+^*} \int\limits_{\partial D} \big[u_t v \nu_0 - t^m u_{x_1} v \nu_1 -$$

$$- t^m u_{x_2} v \nu_2\big] ds + \sup_{v \in E^*} \|v\|^{-1}_{W_+^*} \int\limits_D \big[- u_t v_t + t^m (u_{x_1} v_{x_1} + u_{x_2} v_{x_2}) +$$

$$+ a_1 u_{x_1} v + a_2 u_{x_2} v + a_3 u_t v + a_4 u v\big] dD = \sup_{v \in E^*} \|v\|^{-1}_{W_+^*} \int\limits_{\partial D} \frac{\partial u}{\partial N} v ds +$$

$$+\sup_{v\in E^*}\|v\|^{-1}_{W^*_+}\int\limits_D\big[-u_tv_t+t^m(u_{x_1}v_{x_1}+u_{x_2}v_{x_2})+a_1u_{x_1}v+a_2u_{x_2}v+$$

$$+a_3u_tv+a_4uv\big]dD=\sup_{v\in E^*}\|v\|^{-1}_{W^*_+}\int\limits_D\big[-u_tv_t+t^m(u_{x_1}v_{x_1}+$$

$$+u_{x_2}v_{x_2})+a_1u_{x_1}v+a_2u_{x_1}v+a_3u_tv+a_4uv\big]dD. \tag{9}$$

Due to (6) as well as the Cauchy inequality, we have

$$\Big|\int\limits_D\big[-u_tv_t+t^m(u_{x_1}v_{x_1}+u_{x_2}v_{x_2})\big]dD\Big|\le\Big[\int\limits_D(u_t^2+t^mu_{x_1}^2+$$

$$+t^mu_{x_2}^2)dD\Big]^{\frac12}\times\Big[\int\limits_D(v_t^2+t^mv_{x_1}^2+t^mv_{x_2}^2)dD\Big]^{\frac12}\le\|u\|_{W_+}\|v\|_{W^*_+}, \tag{10}$$

$$\Big|\int\limits_D[a_1u_{x_1}v+a_2u_{x_2}v+a_3u_tv+a_4uv)\big]dD\le$$

$$\le M_1\Big(\int\limits_D t^mu_{x_1}^2dD\Big)^{\frac12}\|v\|_{L_2(D)}+M_2\Big(\int\limits_D t^mu_{x_2}^2dD\Big)^{\frac12}\|v\|_{L_2(D)}+$$

$$+\sup_{\overline{D}}|a_3|\;\|u_t\|_{L_2(D)}\|v\|_{L_2(D)}+\sup_{\overline{D}}|a_4|\;\|u\|_{L_2(D)}\|v\|_{L_2(D)}\le$$

$$\le\Big(\sum_{i=1}^2\big(M_i+\sup_{\overline{D}}|a_{2+i}|\big)\Big)\|u\|_{W_+}\|v\|_{W^*_+}=\widetilde{c}\|u\|_{W_+}\|v\|_{W^*_+}. \tag{11}$$

From (9)–(11) it follows that

$$\|Lu\|_{W^*_-}\le(1+\widetilde{c})\sup_{v\in E^*}\|v\|^{-1}_{W^*_+}\|u\|_{W_+}\|v\|_{W^*_+}=c_1\|u\|_{W_+},$$

i.e., we get inequality (7). Since the proof of inequality (8) repeats that of inequality (7), therefore Lemma 1 is proved completely. □

Remark 2. By virtue of inequality (7) ((8)), the operator $L:W_+\to W^*_-(L^*:W^*_+\to W_-)$ with a dense domain of definition $E(E^*)$ admits a closure, being a continuous operator from the space $W_+(W^*_+)$ to the space $W_-(W^*_-)$. Retaining for this operator the previous notation $L(L^*)$, we note that it is defined on the whole Hilbert space $W_+(W^*_+)$.

Lemma 2. *Problem* (1), (2) *and problem* (3), (4) *are self-conjugate, i.e., for any $u\in W_+$ and $v\in W^*_+$ the following equality holds:*

$$(Lu,v)=(u,L^*v). \tag{12}$$

Proof. According to Remark 2, it suffices to prove equality (12) in the case where $u \in E$ and $v \in E^*$. Obviously, in that case $(Lu, v) = (Lu, v)_{L_2(D)}$. Therefore we have

$$(Lu,v) = (Lu,v)_{L_2(D)} = \int\limits_{\partial D} [u_t v\nu_0 - t^m u_{x_1} v\nu_1 - t^m u_{x_2} v\nu_2] ds +$$

$$+ \int\limits_{\partial D} [a_1\nu_1 + a_2\nu_2 + a_3\nu_0] uv\, ds + \int\limits_{D} \big[- u_t v_t + t^m u_{x_1} v_{x_1} + t^m u_{x_2} v_{x_2} -$$

$$-u(a_1 v)_{x_1} - u(a_2 v)_{x_2} - u(a_3 v)_t + a_4 uv\big] dD = \int\limits_{\partial D} [u_t v\nu_0 - t^m u_{x_1} v\nu_1 -$$

$$-t^m u_{x_2} v\nu_2] ds + \int\limits_{\partial D} [a_1\nu_1 + a_2\nu_2 + a_3\nu_0] uv\, ds - \int\limits_{\partial D} [uv_t\nu_0 -$$

$$-t^m uv_{x_1}\nu_1 - t^m uv_{x_2}\nu_2] ds + \int\limits_{D} \big[uv_{tt} - ut^m v_{x_1x_1} - ut^m v_{x_2x_2} -$$

$$-u(a_1 v)_{x_1} - u(a_2 v)_{x_2} - u(a_3 v)_t + a_4 uv\big] dD = \int\limits_{\partial D} \Big[\Big(v\frac{\partial u}{\partial N} - u\frac{\partial v}{\partial N}\Big) +$$

$$+(u_1\nu_1 + a_2\nu_2 + a_3\nu_0)uv\Big] ds + (u, L^* v)_{L_2(D)}. \tag{13}$$

Equality (12) follows immediately from equalities (2), (4), (5), and (13). □

Consider the conditions

$$\Omega|_S \le 0, \quad \big[t\Omega_t - (\lambda t + m)\Omega\big]\big|_D \ge 0, \tag{14}$$

where the second inequality holds for sufficiently large λ, and $\Omega = a_{1x_1} + a_{2x_2} + a_{3t} - a_4$.

Remark 3. It can be easily seen that inequality (14) is the corollary of the condition

$$\Omega\big|_{\overline{D}} \le \text{const} < 0.$$

Lemma 3. *Let conditions* (6) *and* (14) *be fulfilled. Then for any* $u \in W_+$ *the inequality*

$$c\big\|t^{\frac{1}{2}(m-1)}u\big\|_{L_2(D)} \le \|Lu\|_{W_-^*} \tag{15}$$

with the positive constant c *independent of* u *is valid.*

Proof. Due to Remark 2, it suffices to prove inequality (15) in the case where $u \in E$. If $u \in E$, then for $\alpha = \text{const} > 0$ and $\lambda = \text{const} > 0$ the function

$$v(x_1, x_2, t) = \int\limits_0^t e^{\lambda\tau} \tau^\alpha u(x_1, x_2, \tau) d\tau \tag{16}$$

belongs to the space E^*. The fact that for $\alpha \geq 1$ the function $v \in E^*$ can be easily verified, and for $0 < \alpha < 1$ this statement follows from the well-known Hardy's inequality

$$\int\limits_0^1 t^{-2} g^2(t) dt \leq 4 \int\limits_0^1 f^2(t) dt,$$

where $f(t) \in L_2(0,1)$ and $g(t) = \int_0^t f(\tau) d\tau$.

By (16), the inequalities

$$v_t(x_1, x_2, t) = e^{\lambda t} t^\alpha u(x_1, x_2, t), \quad u(x_1, x_2, t) = e^{-\lambda t} t^{-\alpha} v_t(x_1, x_2, t) \tag{17}$$

are valid.

With regard for (2), (4), (5), and (17) we have

$$\begin{gathered}(Lu, v)_{L_2(D)} = \int\limits_{\partial D} \Big[v \frac{\partial u}{\partial N} + (a_1\nu_1 + a_2\nu_2 + a_3\nu_0) uv \Big] ds + \int\limits_D [-u_t v_t + \\ + t^m u_{x_1} v_{x_1} + t^m u_{x_2} v_{x_2} - u(a_1 v)_{x_1} - u(a_2 v)_{x_2} - u(a_3 v)_t + a_4 uv] dD = \\ = - \int\limits_D e^{\lambda t} t^\alpha u u_t \, dD + \int\limits_D e^{-\lambda t} t^{-\alpha} [t^m (v_{x_1 t} v_{x_1} + v_{x_2 t} v_{x_2}) - \\ - (a_1 v_{x_1} + a_2 v_{x_2}) v_t - (a_{1x_1} + a_{2x_2} + a_{3t} - a_4) v_t v - a_3 v_t^2] dD. \end{gathered} \tag{18}$$

By virtue of (2) we find that

$$\begin{gathered} - \int\limits_D e^{-\lambda t} t^\alpha u u_t \, dD = -\frac{1}{2} \int\limits_D e^{\lambda t} t^\alpha (u^2)_t dt = -\frac{1}{2} \int\limits_{\partial D} e^{\lambda t} t^\alpha u^2 \nu_0 \, ds + \\ + \frac{1}{2} \int\limits_D e^{\lambda t} (\alpha t^{\alpha - 1} + \lambda t^\alpha) u^2 \, dD = \frac{1}{2} \int\limits_D e^{\lambda t} (\alpha t^{\alpha-1} + \lambda t^\alpha) u^2 \, dD = \\ = \frac{\alpha}{2} \int\limits_D e^{\lambda t} t^{\alpha - 1} u^2 \, dD + \frac{1}{2} \int\limits_D \lambda e^{-\lambda t} t^{-\alpha} v_t^2 \, dD, \end{gathered} \tag{19}$$

$$\int\limits_D e^{-\lambda t} t^{m-\alpha} (-v_{x_1 t} v_{x_1} + v_{x_2 t} v_{x_2}) dD = \frac{1}{2} \int\limits_{\partial D} e^{-\lambda t} t^{m-\alpha} (v_{x_1}^2 +$$

$$+v_{x_2}^2)\nu_0\,ds+\frac{1}{2}\int\limits_D e^{-\lambda t}\big[\lambda t^{m-\alpha}+(\alpha-m)t^{m-\alpha-1}\big](v_{x_1}^2+v_{x_2}^2)dD\geq$$

$$\geq\frac{1}{2}\int\limits_D e^{-\lambda t}\big[\lambda t^{m-\alpha}+(\alpha-m)t^{m-\alpha-1}\big](v_{x_1}^2+v_{x_2}^2)dD. \tag{20}$$

In deriving inequality (20) we have taken into account that

$$\nu_0|_S\geq 0,\quad (v_{x_1}^2+v_{x_2}^2)|_{S_0}=0.$$

From (19) we have

$$-\int\limits_D e^{\lambda t}t^{\alpha}uu_t\,dD\geq\frac{\alpha}{2}\big\|t^{\frac{1}{2}(\alpha-1)}u\big\|_{L_2(D)}^2+\frac{1}{2}\int\limits_D \lambda e^{-\lambda t}t^{-\alpha}v_t^2\,dD. \tag{21}$$

Below we assume that the parameter $\alpha=m$.

By (6) we obtain

$$\Big|\int\limits_D e^{-\lambda t}t^{-m}(a_1v_{x_1}+a_2v_{x_2})v_t\,dD\Big|\leq M\int\limits_D e^{-\lambda t}t^{-m}\Big[v_t^2+\frac{1}{2}t^m(v_{x_1}^2+$$

$$+v_{x_2}^2)\Big]dD\leq M\int\limits_D e^{-\lambda t}t^{-m}v_t^2dD+\frac{M}{2}\int\limits_D e^{-\lambda t}(v_{x_1}^2+v_{x_2}^2)dD, \tag{22}$$

where $M=\max(M_1,M_2)$.

Since $\nu_0\big|_S\geq 0$, using conditions (4) and (14) and integrating them by parts, we obtain

$$-\int\limits_D e^{-\lambda t}t^{-m}(a_{1x_1}+a_{2x_2}+a_{3t}-a_4)v_tv\,dD=$$

$$-\frac{1}{2}\int\limits_D e^{-\lambda t}t^{-m}\Omega(v^2)_tdD=-\frac{1}{2}\int\limits_{\partial D} e^{-\lambda t}t^{-m}\Omega v^2\nu_0ds+$$

$$+\frac{1}{2}\int\limits_D e^{-\lambda t}t^{-m-1}\big[t\Omega_t-(\lambda t+m)\Omega\big]v^2dD\geq 0. \tag{23}$$

In deriving inequality (23) we have used the fact that the function $t^{-m}v^2$ has on S_0 a zero trace, i.e., $t^{-m}v^2\big|_{S_0}=0$.

From (18) by virtue of (20)–(23) we have

$$(Lu,v)_{L_2(D)}\geq\frac{m}{2}\big\|t^{\frac{1}{2}(m-1)}u\big\|_{L_2(D)}^2+\frac{1}{2}\int\limits_D \lambda e^{-\lambda t}t^{-m}v_t^2dD+$$

$$+\frac{1}{2}\int\limits_D \lambda e^{-\lambda t}(v_{x_1}^2+v_{x_2}^2)dD-M\int\limits_D e^{-\lambda t}t^{-m}v_t^2dD-\frac{M}{2}\int\limits_D e^{-\lambda t}(v_{x_1}^2+$$

$$+v_{x_2}^2)dD - \sup_{\overline{D}} |a_3| \int_D e^{-\lambda t} t^{-m} v_t^2 dD = \frac{m}{2} \big\| t^{\frac{1}{2}(m-1)} u \big\|_{L_2(D)}^2 +$$

$$+\Big(\frac{\lambda}{2} - M - \sup_{\overline{D}} |a_3|\Big) \int_D e^{-\lambda t} t^{-m} v_t^2 dD + \frac{1}{2}(\lambda - M) \int_D e^{-\lambda t} (v_{x_1}^2 +$$

$$+v_{x_2}^2)dD \geq \frac{m}{2} \big\| t^{\frac{1}{2}(m-1)} u \big\|_{L_2(D)}^2 + \sigma \int_D e^{-\lambda t} (v_t^2 + v_{x_1}^2 + v_{x_2}^2) dD \geq$$

$$\geq \sqrt{2m\sigma \inf_{\overline{D}} e^{-\lambda t}} \big\| t^{\frac{1}{2}(m-1)} u \big\|_{L_2(D)}^2 \Big(\int_D \big[v_t^2 + t^m (v_{x_1}^2 + v_{x_2}^2) \big] dD \Big)^{\frac{1}{2}}, \tag{24}$$

where $\sigma = \big[\frac{\lambda}{2} - M - \sup_{\overline{D}} |a_3|\big] > 0$ for sufficiently large λ, and $\inf_{\overline{D}} e^{-\lambda t} = e^{-\lambda} > 0$. When deriving inequality (24), we have taken into account the fact that $t^{-m}\big|_D \geq 1$.

If $u \in W_+(W_+^*)$ and because $u|_S = 0$ $(u|_{S_0} = 0)$, we can easily prove the inequality

$$\int_D u^2 dD \leq c_0 \int_D u_t^2 dD$$

for which $c_0 = \text{const} > 0$ independent of u. Hence we find that in the space $W_+(W_+^*)$ the norm

$$\|u\|_{W_+(W_+^*)}^2 = \int_D \big[u_t^2 + t^m (u_{x_1}^2 + u_{x_2}^2) + u^2 \big] dD$$

is equivalent to the norm

$$\|u\|^2 = \int_D \big[u_t^2 + t^m (u_{x_1}^2 + u_{x_2}^2) \big] dD. \tag{25}$$

Therefore, retaining for norm (25) the previous designation $\|u\|_{W_+(W_+^*)}$, from (24) we have

$$(Lu, v)_{L_2(D)} \geq \sqrt{2m\sigma e^{-\lambda}} \|t^{\frac{1}{2}(m-1)} u\|_{L_2(D)} \, \|v\|_{W_+^*}. \tag{26}$$

If now we apply the generalized Schwarz inequality

$$(Lu, v) \leq \|Lu\|_{W_-^*} \, \|v\|_{W_+^*}$$

to the left-hand side of (26), then after reducing by $\|v\|_{W_+^*}$ we get inequality (15) in which $c = \sqrt{2m\sigma e^{-\lambda}}$. $\square$

Consider the conditions

$$a_4\big|_{S_0} \geq 0, \quad (\lambda a_4 + a_{4t})\big|_D \geq 0, \tag{27}$$

of which the second one takes place for sufficiently large λ.

Lemma 4. *Let conditions* (6) *and* (27) *be fulfilled. Then for any* $v \in W_+^*$ *the inequality*

$$c\|v\|_{L_2(D)} \leq \|L^*v\|_{W_-} \tag{28}$$

is valid for some $c = \text{const} > 0$ *independent of* $v \in W_+^*$.

Proof. Just as in Lemma 3 and because of Remark 2, it suffices to prove the validity of inequality (28) for $v \in E^*$. Let $v \in E^*$ and let us introduce into the consideration the function

$$u(x_1,x_2,t) = \int\limits_t^{\varphi(x_1,x_2)} e^{-\lambda\tau} v(x_1,x_2,\tau)d\tau, \quad \lambda = \text{const} > 0, \tag{29}$$

where $t = \varphi(x_1,x_2)$ is the equation of the characteristic conoid S. Although on the circle $r = \frac{2}{2+m}$ the function

$$\varphi(x_1,x_2) = \Big[1 - \frac{2+m}{2}r\Big]^{\frac{2}{2+m}}$$

has singularities and at the origin $x_1 = x_2 = 0$, but by the definition of the space E^*, the function v vanishes in some neighborhood of the circle $\Gamma = S \cap S_0$ and of the segment $I: x_1 = x_2 = 0,\ 0 \leq t \leq 1$, the function u defined by equality (29) will belong to the space E. Moreover, it is obvious that the equalities

$$u_t(x_1,x_2,t) = -e^{-\lambda t}v(x_1,x_2,t), \quad v(x_1,x_2,t) = -e^{\lambda t}u_t(x_1,x_2,t) \tag{30}$$

hold.

Owing to (2), (4), (5), and (30), we have

$$\begin{aligned}
(L^*v,u)_{L_2(D)} &= \int\limits_{\partial D} \Big[u\frac{\partial v}{\partial N} - (a_1\nu_1 + a_2\nu_2 + a_3\nu_0)vu\Big]ds + \\
&+ \int\limits_D [-v_t u_t + t^m v_{x_1}u_{x_1} + t^m v_{x_2}u_{x_2} + a_1 v u_{x_1} + a_2 v u_{x_2} + a_3 v u_t + \\
&+ a_4 uv]dD = \int\limits_D e^{-\lambda t} v_t v\, dD - \int\limits_D e^{\lambda t}\big[t^m(u_{x_1t}u_{x_1} + u_{x_2t}u_{x_2}) + \\
&+ (a_1u_{x_1} + a_2u_{x_2})u_t + a_3u_t^2 + a_4uu_t\big]\, dD,
\end{aligned} \tag{31}$$

$$\int\limits_D e^{-\lambda t} v_t v \, dD = \frac{1}{2} \int\limits_{\partial D} e^{-\lambda t} v^2 \nu_0 \, ds + \frac{1}{2} \int\limits_D e^{-\lambda t} \lambda v^2 \, dD =$$

$$= \frac{1}{2} \int\limits_S e^{-\lambda t} v^2 \nu_0 \, ds + \frac{1}{2} \int\limits_D e^{-\lambda t} \lambda v^2 \, dD =$$

$$= \frac{1}{2} \int\limits_S e^{\lambda t} u_t^2 \nu_0 \, ds + \frac{1}{2} \int\limits_D e^{\lambda t} \lambda u_t^2 \, dD, \tag{32}$$

$$-\int\limits_D e^{\lambda t} t^m (u_{x_1 t} u_{x_1} + u_{x_2 t} u_{x_2}) dD = -\frac{1}{2} \int\limits_{\partial D} e^{\lambda t} t^m (u_{x_1}^2 + u_{x_2}^2) \nu_0 \, ds +$$

$$+\frac{1}{2} \int\limits_D e^{\lambda t} [\lambda t^m + m t^{m-1}] (u_{x_1}^2 + u_{x_2}^2) dD \geq$$

$$\geq -\frac{1}{2} \int\limits_{\partial D} e^{\lambda t} t^m (u_{x_1}^2 + u_{x_2}^2) \nu_0 \, ds + \frac{1}{2} \int\limits_D \lambda e^{\lambda t} t^m (u_{x_1}^2 + u_{x_2}^2) \, dD. \tag{33}$$

Since $u|_S = 0$, for some α we have $v_t = \alpha \nu_0$, $v_{x_1} = \alpha \nu_1$, $v_{x_2} = \alpha \nu_2$ on S. Therefore the fact that the surface S is characteristic results in

$$\left[u_t^2 - t^m (u_{x_1}^2 + u_{x_2}^2)\right]\Big|_S = \alpha^2 \left[\nu_0^2 - t^m (\nu_1^2 + \nu_2^2)\right]\Big|_S = 0. \tag{34}$$

Taking into account that $m > 0$ and hence $t^m|_{S_0} = 0$, equalities (2) and (34) imply

$$\frac{1}{2} \int\limits_S e^{\lambda t} u_t^2 \nu_0 \, ds - \frac{1}{2} \int\limits_{\partial D} e^{\lambda t} t^m (u_{x_1}^2 + u_{x_2}^2) \nu_0 \, ds =$$

$$= \frac{1}{2} \int\limits_S e^{\lambda t} [u_t^2 - t^m (u_{x_1}^2 + u_{x_2}^2)] \nu_0 \, ds = 0. \tag{35}$$

Due to (32), (33), and (35), equality (31) yields

$$(L^* v, u)_{L_2(D)} \geq \frac{1}{2} \int\limits_D e^{\lambda t} \left[u_t^2 + t^m (u_{x_1}^2 + u_{x_2}^2)\right] dD -$$

$$-\int\limits_D e^{\lambda t} \left[(a_1 u_{x_1} + a_2 u_{x_2}) u_t + a_3 u_t^2 + a_4 u u_t\right] dD. \tag{36}$$

Since $\nu_0\big|_{S_0} < 0$, by (27) we have

$$-\int\limits_D e^{\lambda t} a_4 u u_t dD = -\frac{1}{2} \int\limits_D e^{\lambda t} a_4 (u^2)_t dD = -\frac{1}{2} \int\limits_{\partial D} e^{\lambda t} a_4 u^2 \nu_0 ds +$$

$$+\frac{1}{2}\int\limits_D e^{\lambda t}(\lambda a_4 + a_{4t})u^2 dD \geq 0. \tag{37}$$

Using (6), we obtain

$$\Big|\int\limits_D e^{\lambda t}(a_1 u_{x_1} + a_2 u_{x_2})u_t dD\Big| \leq M\int\limits_D e^{\lambda t}\Big[u_t^2 + \frac{1}{2}t^m(u_{x_1}^2 + u_{x_2}^2)\Big]dD, \tag{38}$$

where $M = \max(M_1, M_2)$.

With regard for (30), (37) and (38), from (36) we get

$$(L^*v,u)_{L_2(D)} \geq \Big(\frac{\lambda}{2} - M - \sup_{\overline{D}}|a_3|\Big)\int\limits_D e^{\lambda t}\big[u_t^2 + t^m(u_{x_1}^2 + u_{x_2}^2)\big]dD \geq$$

$$\geq \gamma\Big[\int\limits_D e^{\lambda t}u_t^2 dD\Big]^{\frac{1}{2}}\Big[\int\limits_D e^{\lambda t}\big[u_t^2 + t^m(u_{x_1}^2 + u_{x_2}^2)\big]dD\Big]^{\frac{1}{2}} =$$

$$= \gamma\Big[\int\limits_D e^{-\lambda t}v^2 dD\Big]^{\frac{1}{2}}\Big[\int\limits_D e^{\lambda t}\big[u_t^2 + t^m(u_{x_1}^2 + u_{x_2}^2)\big]dD\Big]^{\frac{1}{2}} \geq$$

$$\geq \gamma\inf_{\overline{D}} e^{-\lambda t}\|v\|_{L_2(D)}\Big[\int\limits_D \big[u_t^2 + t^m(u_{x_1}^2 + u_{x_2}^2)\big]dD\Big]^{\frac{1}{2}}, \tag{39}$$

where $\gamma = \big(\frac{\lambda}{2} - M - \sup_{\overline{D}}|a_3|\big) > 0$ for sufficiently large λ.

From (39), in just the same way as in obtaining inequality (26), we find that

$$(L^*v,u)_{L_2(D)} \geq c\|v\|_{L_2(D)}\,\|u\|_{W_+},$$

which immediately implies (28). $\square$

Denote by $L_{2,\alpha}(D)$ a space of functions u such that $t^\alpha u \in L_2(D)$. Assume

$$\|u\|_{L_{2,\alpha}(D)} = \|t^a lu\|L_2(D), \quad \alpha_m = \frac{1}{2}(m-1).$$

Definition 1. For $F \in W_-^*$ we call the function u a strong generalized solution of problem (1), (2) of the class L_{2,α_m}, if $u \in L_{2,\alpha_m}(D)$ and there exists a sequence of functions $u_n \in E$ such that $u_n \to u$ in the space $L_{2,\alpha_m}(D)$ and $Lu_n \to F$ in the space W_-^* as $n \to \infty$, i.e.,

$$\lim_{n\to\infty}\|u_n - u\|_{L_{2,\alpha_m}(D)} = 0, \quad \lim_{n\to\infty}\|Lu_n - F\|_{W_-^*} = 0.$$

Definition 2. For $F \in L_2(D)$ we call the function u a strong generalized solution of problem (1), (2) of the class W_+, if $u \in W_+$ and there exists a sequence of functions $u_n \in E$ such that $u_n \to u$ and $Lu_n \to F$ in the spaces W_+ and W_-^*, respectively, i.e.,

$$\lim_{n\to\infty} \|u_n - u\|_{W_+} = 0, \quad \lim_{n\to\infty} \|Lu_n - F\|_{W_-^*} = 0.$$

According to the results obtained in [13], the following theorems are the corollaries of Lemmas 1–4.

Theorem 1. *Let conditions* (6), (14), *and* (27) *be fulfilled. Then for every $F \in W_-^*$ there exists a unique strong generalized solution u of problem* (1), (2) *of the class L_{2,α_m} for which the estimate*

$$\|u\|_{L_{2,\alpha_m}(D)} \leq c\|F\|_{W_-^*} \tag{40}$$

with the constant c independent of F is valid.

Theorem 2. *Let conditions* (6), (14), *and* (27) *be fulfilled. Then for every $F \in L_2(D)$ there exists a unique strong generalized solution u of problem* (1), (2) *of the class W_+ for which estimate* (40) *is valid.*

Consider now a second-order hyperbolic equation with a characteristic degeneration of the kind

$$L_1 u \equiv (t^m u_t)_t - u_{x_1x_1} - u_{x_2x_2} + a_1 u_{x_1} + a_2 u_{x_2} + a_3 u_t + a_4 u = F, \tag{41}$$

where $1 \leq m = \text{const} < 2$.

Denote by

$$D_1 : 0 < t < \Big[1 - \frac{2-m}{2} r\Big]^{\frac{2}{2-m}}, \quad r = (x_1^2 + x_2^2)^{\frac{1}{2}} < \frac{2}{2-m}$$

a bounded domain lying in a half-space $t > 0$, bounded above by the characteristic conoid

$$S_2 : t = \Big[1 - \frac{2-m}{2} r\Big]^{\frac{2}{2-m}}, \quad r \leq \frac{2}{2-m}$$

of equation (41) with the vertex at the point $(0,0,1)$ and below by the base

$$S_1 : t = 0, \quad r \leq \frac{2}{2-m}$$

of the same conoid where equation (41) has a characteristic degeneration. Just as in the case of equation (1), in what follows, the coefficients a_i, $i = 1, \dots, 4$, of equation (41) in the domain D_1 are assumed to be the functions of the class $C^2(\overline{D})$.

For equation (41) let us consider a multidimensional version of the characteristic problem which is formulated as follows: Find in the domain D_1 a solution $u(x_1, x_2, t)$ of equation (41) satisfying the boundary condition

$$u\big|_{S_1} = 0 \tag{42}$$

on the plane characteristic surface S_1.

The characteristic problem for the equation

$$L_1^* v \equiv (t^m v_t)_t - v_{x_1x_1} - v_{x_2x_2} - (a_1 v)_{x_1} - (a_2 v)_{x_2} - (a_3 v)_t + a_4 v = F \quad (43)$$

in the domain D_1 is formulated analogously for the boundary condition

$$v\big|_{S_2} = 0, \quad (44)$$

where L_1^* is the operator conjugate formally to the operator L_1.

Denote by E_1 and E_1^* the classes of functions from the Sobolev space $W_2^2(D_1)$, satisfying the corresponding boundary condition (42) or (44) and vanishing in some (own for every function) three-dimensional neighborhood of the segment $I : x_1 = 0,\ x_2 = 0,\ 0 \le t \le 1$. Let $W_{1+}(W_{1+}^*)$ be the Hilbert space obtained by closing the space $E_1(E_1^*)$ in the norm

$$\|u\|^2 = \int\limits_{D_1} [u_t^2 + u_{x_1}^2 + u_{x_2}^2 + u^2] dD_1.$$

Denote by $W_{1-}(W_{1-}^*)$ a space with a negative norm, constructed with respect to $L_2(D_1)$ and $W_{1+}(W_{1+}^*)$.

The following lemma can be proved analogously to Lemmas 1 and 2.

Lemma 5. *For all functions $u \in E_1$ and $v \in E_1^*$ the inequalities*

$$\|L_1 u\|_{W_{1-}^*} \le c_1 \|u\|_{W_{1+}}, \quad \|L_1^* v\|_{W_{1-}} \le c_1 \|v\|_{W_{1+}^*},$$

are fulfilled and problems (41), (42) *and* (43), (44) *are self-conjugate, i.e., for every $u \in W_{1+}$ and $v \in W_{1+}^*$ the equality*

$$(L_1 u, v) = (u, L_1^* v)$$

holds.

Let us consider the conditions

$$\inf_{\overline{D}_1} (a_4 - a_{1x_1} - a_{2x_2} - a_{3t}) > 0, \quad (45)$$

$$\inf_{S_1} a_3 > \frac{1}{2} \ \text{ for } \ m = 1, \ \ \inf_{S_1} a_3 > 0 \ \text{ for } \ m > 1. \quad (46)$$

Lemma 6. *Let conditions* (45) *and* (46) *be fulfilled. Then for any $u \in W_{1+}$ the inequality*

$$c\|u\|_{L_2(D_1)} \le \|L_1 u\|_{W_{1-}^*}, \quad (47)$$

with the positive constant c independent of u, is valid.

Consider now the conditions

$$\inf_{\overline{D}_1} a_4 > 0, \tag{48}$$

$$\inf_{S_1} a_3 > -\frac{1}{2} \text{ for } m = 1, \ \inf_{S_1} a_3 > 0 \text{ for } m > 1. \tag{49}$$

Note that condition (46) results in condition (49).

Lemma 7. *Let conditions* (48) *and* (49) *be fulfilled. Then for any* $v \in W_{1+}^*$ *the inequality*

$$c\|v\|_{L_2(D_1)} \le \|L_1^* v\|_{W_{1-}} \tag{50}$$

with the positive constant c *independent of* v *holds.*

Below we will restrict ourselves to proving only Lemma 6. Let $u \in E_1$. We introduce into consideration the function

$$v(x_1, x_2, t) = \int\limits_t^{\psi(x_1,x_2)} e^{-\lambda\tau} u(x_1, x_2, \tau) d\tau, \quad \lambda = \text{const} > 0, \tag{51}$$

where $t = \psi(x_1, x_2)$ is the equation of the characteristic conoid S_2 of equation (41). Since $1 \le m < 2$, the first and second-order derivatives of the function

$$\psi(x_1, x_2) = \Big[1 - \frac{2-m}{2} r\Big]^{\frac{2}{2-m}}$$

with respect to the variables x_1 and x_2 will have singularities at the origin only. But by the definition of the space E_1, the function u vanishes in some neighborhood of the segment $I : x_1 = x_2 = 0, \ 0 \le t \le 1$. Therefore the function v defined by equality (51) belongs to the space E_1^*, and the equalities

$$v_t(x_1, x_2, t) = -e^{-\lambda t} u(x_1, x_2, t), \quad u_t(x_1, x_2, t) = -e^{\lambda t} v_t(x_1, x_2, t) \tag{52}$$

hold.

Since the derivative with respect to the conormal

$$\frac{\partial}{\partial N} = t^m \nu_0 \frac{\partial}{\partial t} - \nu_1 \frac{\partial}{\partial x_1} - \nu_2 \frac{\partial}{\partial x_2}$$

for the operator L_1 is an interior differential operator on the characteristic surfaces of equation (41), because of (42) and (44) for the functions $u \in E_1$ and $v \in E_1^*$ we have

$$\frac{\partial u}{\partial N}\Big|_{S_1} = 0, \quad \frac{\partial u}{\partial N}\Big|_{S_2} = 0. \tag{53}$$

By (42), (44), (52) and (53) we arrive at

$$(Lu,v)_{L_2(D_1)}=\int\limits_{\partial D_1}\Big[v\frac{\partial u}{\partial N}+(a_1\nu_1+a_2\nu_2+a_3\nu_0)uv\Big]ds+$$

$$+\int\limits_{D_1}\big[-t^m u_t v_t+u_{x_1}v_{x_1}+u_{x_2}v_{x_2}-u(a_1v)_{x_1}-u(a_2v)_{x_2}-u(a_3v)_t+$$

$$+a_4uv\big]dD_1=\int\limits_{D_1}e^{-\lambda t}t^m uu_t dD_1+\int\limits_{D_1}e^{\lambda t}\big[-v_{x_1t}v_{x_1}-v_{x_2t}v_{x_2}+$$

$$+(a_1v_{x_1}+a_2v_{x_2})v_t+(a_{1x_1}+a_{2x_2}+a_{3t}-a_4)v_tv++a_3v_t^2\big]dD,\qquad(54)$$

$$\int\limits_{D_1}e^{-\lambda t}t^m uu_t dD_1=\frac{1}{2}\int\limits_{D_1}e^{-\lambda t}t^m(u^2)_t dD_1=\frac{1}{2}\int\limits_{\partial D_1}e^{-\lambda t}t^m u^2\nu_0 ds+$$

$$+\frac{1}{2}\int\limits_{D_1}e^{-\lambda t}(\lambda t^m-mt^{m-1})u^2dD_1=\frac{1}{2}\int\limits_{S_2}e^{-\lambda t}t^m u^2\nu_0 ds+$$

$$+\frac{1}{2}\int\limits_{D_1}e^{\lambda t}(\lambda t^m-mt^{m-1})v_t^2dD_1=\frac{1}{2}\int\limits_{S_2}e^{\lambda t}t^m v_t^2\nu_0 ds+$$

$$+\frac{1}{2}\int\limits_{D_1}e^{\lambda t}(\lambda t^m-mt^{m-1})v_t^2dD_1,\qquad(55)$$

$$\int\limits_{D_1}e^{\lambda t}[-v_{x_1t}v_{x_1}-v_{x_2t}v_{x_2}]dD_1=-\frac{1}{2}\int\limits_{\partial D_1}e^{\lambda t}[v_{x_1}^2+v_{x_2}^2]\nu_0 ds+$$

$$+\frac{1}{2}\int\limits_{D_1}e^{\lambda t}\lambda[v_{x_1}^2+v_{x_2}^2]dD_1.\qquad(56)$$

Since $v\big|_{S_2}=0$ and the surface S_2 is characteristic, similarly to equality (34) we have

$$(t^m v_t^2-v_{x_1}^2-v_{x_2}^2)\big|_{S_2}=0.\qquad(57)$$

Taking into account that $\nu_0\big|_{S_1}<0$, with regard for equalities (54)–(57) we find that

$$(Lu,v)_{L_2(D)}=-\frac{1}{2}\int\limits_{S_1}e^{\lambda t}[v_{x_1}^2+v_{x_2}^2]\nu_0 ds+\frac{1}{2}\int\limits_{S_2}e^{\lambda t}[t^m v_t^2-v_{x_1}^2-$$

$$-v_{x_2}^2]\nu_0 ds+\frac{1}{2}\int\limits_{D_1}e^{\lambda t}[2a_3-mt^{m-1}+\lambda t^m]v_t^2dD_1+\frac{1}{2}\int\limits_{D_1}e^{\lambda t}\lambda[v_{x_1}^2+$$

$$+v_{x_2}^2]dD_1+\int\limits_{D_1} e^{\lambda t}[a_1v_{x_1}+a_2v_{x_2}]v_t dD_1+\int\limits_{D_1}(a_{1x_1}+a_{2x_2}+a_{3t}-$$

$$-a_4)v_t v dD_1 \geq \frac{1}{2}\int\limits_{D_1} e^{\lambda t}[2a_3-mt^{m-1}+\lambda t^m]v_t^2 dD_1+$$

$$+\frac{1}{2}\int\limits_{D_1} e^{\lambda t}\lambda[v_{x_1}^2+v_{x_2}^2]dD_1-\Big|\int\limits_{D_1} e^{\lambda t}[a_1v_{x_1}+a_2v_{x_2}]v_t dD_1+$$

$$+\int\limits_{D_1} e^{\lambda t}(a_{1x_1}+a_{2x_2}+a_{3t}-a_4)v_t v dD_1. \qquad (58)$$

Since $a_3 \in C(\overline{D})$, it follows from condition (46) that for sufficiently large λ

$$(2a_3-mt^{m-1}+\lambda t^m\big|_{D_1} \geq 4\delta=\text{const}>0$$

and thus

$$\frac{1}{2}\int\limits_{D_1} e^{\lambda t}[2a_3-mt^{m-1}+\lambda t^m]v_t^2 dD_1 \geq 2\delta\int\limits_{D_1} e^{\lambda t}v_t^2 dD_1. \qquad (59)$$

Integration by parts gives

$$\int\limits_{D_1} e^{\lambda t}(a_{1x_1}+a_{2x_2}+a_{3t}-a_4)v_t v dD_1=\frac{1}{2}\int\limits_{\partial D_1} e^{\lambda t}(a_{1x_1}+a_{2x_2}+$$

$$+a_{3t}-a_4)v^2\nu_0 ds-\frac{1}{2}\int\limits_{D_1} e^{\lambda t}\big[\lambda(a_{1x_1}+a_{2x_2}+a_{3t}-a_4)+$$

$$+(a_{1x_1}+a_{2x_2}+a_{3t}-a_4)_t\big]v^2 dD_1,$$

whence by condition (44) and inequalities $\nu_0\big|_{S_1}<0$ and (45) we find that for sufficiently large λ the inequality

$$\int\limits_{D_1} e^{\lambda t}(a_{1x_1}+a_{2x_2}+a_{3t}-a_4)v_t v dD_1 \geq 0 \qquad (60)$$

is valid.

Using the inequality

$$(a+b)^2 \leq 2a^2+2b^2, \quad |ab| \leq \delta|a|^2+\frac{1}{4\delta}|b|^2,$$

we obtain

$$\Big|\int\limits_{D_1} e^{\lambda t}[a_1v_{x_1}+a_2v_{x_2}]v_t dD_1 \leq \delta\int\limits_{D_1} e^{\lambda t}v_t^2 dD_1+$$

$$+\frac{\gamma}{2\delta}\int\limits_{D_1} e^{\lambda t}(v_{x_1}^2+v_{x_2}^2)dD_1, \tag{61}$$

where $\gamma=\max\big(\sup_{\overline{D}_1}|a_1|^2,\sup_{\overline{D}_1}|a_2|^2\big)$.

With regard for (59), (60), and (61), for sufficiently large λ we get from (58) that

$$(Lu,v)_{L_2(D)}\geq\delta\int\limits_{D_1} e^{\lambda t}v_t^2dD_1+\Big(\frac{\lambda}{2}-\frac{\lambda}{2\delta}\Big)\int\limits_{D_1} e^{\lambda t}(v_{x_1}^2+v_{x_2}^2)dD_1,$$

which for $\lambda\geq 2\delta+\frac{\gamma}{\delta}$ yields

$$(Lu,v)_{L_2(D)}\geq\delta\int\limits_{D_1} e^{\lambda t}[v_t^2+v_{x_1}^2+v_{x_2}^2]dD_1. \tag{62}$$

In the same way as in proving inequality (15) in Lemma 3, from (62) follows inequality (47) which proves Lemma 6.

Definition 3. For $F\in W_{1-}^*(W_{1-})$ we call the function $u(v)$ a strong generalized solution of problem (41), (42) (of problem (43), (44)) of the class L_2, if $u(v)\in L_2(D_1)$ and there exists a sequence of functions $u_n(v_n)\in E_1(E_1^*)$ such that $u_n\to u$ $(v_n\to v)$ in the space $L_2(D_1)$ and $L_1u_n\to F$ $(L_1^*v_n\to F)$ in the space $W_{1-}^*(W_{1-})$ as $n\to\infty$.

Definition 4. For $F\in L_2(D)$ we call the function $u(v)$ a strong generalized solution of problem (41), (42) (of problem (43), (44)) of the class $W_{1+}(W_{1+}^*)$, if $u(v)\in W_{1+}(W_{1+}^*)$ and there exists a sequence of functions $u_n(v_n)\in E_1(E_1^*)$ such that $u_n\to u$ $(v_n\to v)$ and $L_1u_n\to F(L_1^*v_n\to F)$ in the spaces $W_{1+}(W_{1+}^*)$ and $W_{1-}^*(W_{1-})$, respectively.

The following theorems are the corollaries of Lemmas 5–7.

Theorem 3. *Let conditions* (45), (46), *and* (48) *be fulfilled. Then for any* $F\in W_{1-}^*(W_{1-})$ *there exists a unique strong generalized solution* $u(v)$ *of problem* (41), (42) (*of problem* (43), (44)) *of the class* L_2 *for which the estimate*

$$\|u\|_{L_2(D_1)}\leq c\|F\|_{W_{1-}^*}\quad\big(\|v\|_{L_2(D_1)}\leq c\|F\|_{W_{1-}}\big) \tag{63}$$

with the positive constant c *independent of* F *holds.*

Theorem 4. *Let conditions* (45), (46), *and* (48) *be fulfilled. Then for any* $F\in L_2(D_1)$ *there exists a unique strong generalized solution* $u(v)$ *of problem* (41), (42) (*of problem* (43), (44)) *of the class* $W_{1+}(W_{1+}^*)$ *for which estimate* (63) *is valid.*

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