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SEVERAL COHOMOLOGY ALGEBRAS CONNECTED WITH THE POISSON STRUCTURE

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Abstract. The structure of a Lie superalgebra is defined on the space of multiderivations of a commutative algebra. This structure is used to define some cohomology algebra of Poisson structure. It is shown that when a commutative algebra is an algebra of C^∞-functions on the C^∞-manifold, the cohomology algebra of Poisson structure is isomorphic to an algebra of vertical cohomologies of the foliation corresponding to the Poisson structure.

§ 0. Introduction

0.1. Let M be a finite-dimensional C^∞-manifold. We use the following notation: $\Omega^K(M)$, $k=1,2,\dots$, is the $C^\infty(M)$-module of differential k-form on M; $V^k(M)$, $k=1,2,\dots$, is the $C^\infty(M)$-module of contravariant antisymmetric tensor fields of degree k on M; S is some foliation on the manifold M; $V^k(M,S)$, $k=1,2,\dots$, is a submodule of $V^k(M)$ consisting of the fields tangent to the leaves of the foliation S; $\Omega^k(M,S)$, $k=1,2,\dots$, is the $C^\infty(M)$-module of homomorphisms from the module $V^k(M,S)$ into $C^\infty(M)$; $\Omega^k_s(M)$, $k=1,2,\dots$, is a submodule of $\Omega^k(M)$ consisting of k-forms vanishing on $V^k(M,S)$. Also, we put

$$\Omega^0(M)=V^0(M)=\Omega^0(M,S)=V^0(M,S)=C^\infty(M);$$

$$\Omega^*(M)=\overset{\infty}{\underset{k=0}{\oplus}}\Omega^k(M);\quad V^*(M)=\overset{\infty}{\underset{k=0}{\oplus}}V^k(M);$$

$$\Omega^*(M,S)=\overset{\infty}{\underset{k=0}{\oplus}}\Omega^k(M,S);\quad V^*(M,S)=\overset{\infty}{\underset{k=0}{\oplus}}V^k(M,S);\quad \Omega^*_s(M)=\overset{\infty}{\underset{k=0}{\oplus}}\Omega^*_s(M),$$

where $\Omega^0_s(M)$ is a subalgebra of $C^\infty(M)$ consisting of functions constant along the leaves of the foliation S.

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0.2. The exterior derivation $d : \Omega^k(M) \to \Omega^{k+1}(M)$ carries $\Omega_s^k(M)$ into $\Omega_s^{k+1}(M)$ and thus induces a differential $\widetilde{d} : \Omega^k(M)/\Omega_s^k(M) \to \Omega^{k+1}(M)/\Omega_s^{k+1}(M)$.

A cohomology of the complex $(\Omega^*(M)/\Omega_s^*(M), \widetilde{d})$ is called a relative cohomology of the foliated manifold (M,S). It is a generalization of cohomology of the family of manifold defined in [1]. We denote the pth cohomology space by $H^p(M,S)$, and the cohomology algebra $\overset{\infty}{\underset{k=0}{\oplus}} H^p(M,S)$ by $H^*(M,S)$.

0.3. Let $R : \Omega^*(M) \to \Omega^*(M,S)$ be a restriction map. It is clear that Kernel $(R) = \Omega_s^*(M)$. If we denote by d_s the operator of exterior derivation on $\Omega^k(M,S)$, then it can be said that the map R is a homomorphism of the complex $(\Omega^*(M), d)$ into the complex $(\Omega^*(M,S), d_s)$. In general, the homomorphism R is not an epimorphism, and therefore, in general, the induced homomorphism $\widetilde{R} : (\Omega^*(M)/\Omega_s^*(M), \widetilde{d}) \to (\Omega^*(M,S), d_s)$ is not an isomorphism.

If we denote the pth cohomology space of the complex $(\Omega^*(M,S), d_s)$ by $H_s^p(M)$ and the cohomology algebra $\overset{\infty}{\underset{k=0}{\oplus}} H_s^p(M)$ by $H_s^*(M)$, we can say that, in general, the algebras $H^*(M,S)$ and $H_s^*(M)$ are not isomorphic though we have the natural homomorphism $[R] : H^*(M,S) \to H_s^*(M)$ induced by $\widetilde{R}$.

0.4. In the case where the manifold M is provided with a Riemannian metric, we have the map of orthogonal projection $\pi : V'(M) \to V'(M,S)$. The map π induces the endomorphism π^* of the algebra $\Omega^*(M)$ defined as $(\pi^* w)(v_1, \dots, v_k) = w(\pi v_1, \dots, \pi v_k)$. It is clear that π^* is the projection $\pi^* \circ \pi^* = \pi^*$. We denote the subalgebra Image(π^*) by $\Omega_v^*(M)$ and call its elements vertical differential forms on the foliated manifold (M,S) (see [2]).

It is easy to check that the operator $\pi^* \circ d \equiv d_v : \Omega_v^*(M) \to \Omega_v^*(M)$ is a coboundary operator, and we call the cohomology algebra of the complex $(\Omega_v^*(M), d_v)$ the algebra of vertical cohomologies of the foliation S_i and denote it by $H_v^*(M)$ (see [2]).

0.5. If M is a Riemannian manifold, we can define the reverse map of $\widetilde{R}$ as follows: $(R^{-1}w)(v_1, \dots, v_k) = w(\pi v_1, \dots, \pi v_k)$, and $\widetilde{R}^{-1}(w) = [R^{-1}w]$. So, the complexes $(\Omega^*(M)/\Omega_s^*(M), \widetilde{d})$, $(\Omega^*(M,S), d_s)$, and $(\Omega_v^*(M), d_s)$ are isomorphic.

For the foliated Riemannian manifold (M,S), three cohomology algebras $H^*(M,S)$, $H_s^*(M)$, and $H_v^*(M)$ are isomorphic.

0.6. The definition of the complexes $(\Omega^*(M)/\Omega_s^*(M), \widetilde{d})$, $\big(\Omega^k(M,S), d_s\big)$ and $\big(\Omega_v^*(M), d_v\big)$ can be generalized as follows: Let L be a $C^\infty(M)$-submodule of $V'(M)$, and also be a Lie subalgebra of $V'(M)$. Let us denote by $\Omega_L^*(M)$ a subalgebra of the exterior algebra $\Omega^*(M)$ consisting of the forms w such that $w(u_1, \dots, u_n) = 0$ for every system $\{u_1, \dots, u_n\} \subset L$. Further, we denote by $\Omega^*(M,L)$ the algebra of $C^\infty(M)$-multilinear antisymmetric maps from L^k into $C^\infty(M)$.

The definition of derivations $d_L : \Omega^*(M,L) \to \Omega^*(M,L)$ and $\widetilde{d} : \Omega^*(M)/\Omega^*_L(M) \to \Omega^*(M)/\Omega^*_L(M)$ is clear.

Indeed, the cohomologies of the complex $(\Omega^*(M,L), d_L)$ are the cohomologies of the Lie algebra L, with coefficients in $C^\infty(M)$, denoted by $H^*(L, C^\infty(M))$ (see [3]).

In the cases considered in 0.1 – 0.5, the submodule L is $V'(M,S)$.

If there is some projector $\pi : V'(M) \to V'(M)$ with Image $(\pi) = L$, then the algebra of vertical cohomologies can be defined as in 0.4. The proof of the fact that the cohomologies of the complexes

$$(\Omega^*(M)/\Omega^*_L(M), \widetilde{d}), \quad \big(\Omega^*(M,L), d_L\big), \quad \text{and} \quad \big(\Omega^*_v(M), d_v\big)$$

are isomorphic is analogous to the proof of the thereom in 0.5.

We use the above-described generalization in §2 in considering a cohomology of the Poisson structure.

In §1 we introduce the notion of Poisson algebra and define its cohomologies. We also describe here some algebraic constructions which help us to arrange a connection between the cohomologies defined in §0 and the cohomologies of the Poisson structure.

§ 1. Lie Superalgebra Structure on the Space of Multiderivations of a Commutative Algebra. The Poisson Algebra

1.1. Let F be a real or complex vector space. For each positive integer k we denote by $A^k(F)$ the space of multilinear antisymmetric maps from F^k into F. Also we put $A^0(F) = F$ and $A^*(F) = \overset{\infty}{\underset{k=0}{\oplus}} A^k(F)$.

1.2. There is a natural structure of the Lie subalgebra on $A'(F)$ defined by the commutator. It might be defined as a structure of the Lie subalgebra on $A^*(F)$. The supercommutator $[\alpha,\beta] \in A^{m+n-1}(F)$ of two elements $\alpha \in A^m(F)$ and $\beta \in A^n(F)$ is defined as follows (see [4]):

$$\begin{gathered}[\alpha,\beta](v_1,\dots,v_{m+n-1}) = \\ = \frac{1}{m!\,n!}\sum_s \operatorname{sgn}(s)\big((-1)^{mn+n}\alpha(\beta(v_{s(1)},\dots,v_{s(n)}), v_{s(n+1)},\dots \\ \dots, v_{s(m+n-1)}\big) + (-1)^m\beta\big(\alpha(v_{s(1)},\dots,v_{s(n)}), v_{s(n+1)},\dots,v_{s(m+n-1)})\big);\end{gathered}$$

also, for $v, w \in A^0(F) = F$ we put $[\alpha, v](v_1,\dots,v_{m-1}) = [v,\alpha](v_1,\dots,v_{m-1}) = \alpha(v, v_1,\dots,v_{m-1})$ and $[v,w] = 0$.

1.3. It is easy to check that the bracket as defined above satisfies the axioms of the Lie superalgebra: For $\alpha \in A^m(F)$, $\beta \in A^n(F)$ and $\gamma \in A^k(F)$ we have (a) $[\alpha,\beta]=(-1)^{mn}[\beta,\alpha]$; (b) $(-1)^{mk}[[\alpha,\beta],\gamma]+(-1)^{mn}[[\beta,\gamma],\alpha]+(-1)^{nk}[[\gamma,\alpha],\beta]=0$.

1.4. One classical notion that can be translated into the language of the bracket defined in $A^*(F)$ is the notion of "a Lie algebra structure on F". A structure of the Lie algebra on F is an element $\mu \in A^2(F)$ satisfying the condition $[\mu,\mu]=0$. The latter is equivalent to the Jacobi identity

$$\mu\big(\mu(a,b),c\big)+\mu\big(\mu(b,c),d\big)+\mu\big(\mu(c,a),b\big).$$

We call such an element an involutive element.

1.5. An involutive element $\mu \in A^2(F)$ defines the linear operator $\widetilde{\mu}$: $A^*(F)\to A^*(F)$, $\widetilde{\mu}(\alpha)=[\mu,\alpha]$. It is clear that if $\alpha\in A^k(F)$, then $\widetilde{\mu}(\alpha)\in A^{k+1}(F)$. Moreover, the property (b) in 1.3 implies $\widetilde{\mu}^2=0$, i.e., $\widetilde{\mu}$ is a coboundary operator and therefore defines some space of cohomologies. As a matter of fact, it is the Chevalley–Eilenberg cohomology of the Lie algebra F with coefficients in F (see [4]).

1.6. Further we shall consider only the case with F as a commutative algebra over the field of real or complex numbers.

In that case, the space $A^*(F)$ has a structure of the anticommutative (exterior) algebra defined by the classical formula: For $\alpha\in A^m(F)$, $\beta\in A^n(F)$, and $a\in A^0(F)=F$ we have

$$(\alpha\beta)(v_1,\dots,v_{m+n})=$$
$$=\frac{1}{m!\,n!}\sum_s \operatorname{sgn}(s)\alpha(v_{s(1)},\dots,v_{s(m)})\beta(v_{s(m+1)},\dots,v_{s(m+n)}),$$

and $(a\alpha)(v_1,\dots,v_m)=(\alpha a)(v_1,\dots,v_m)=a\cdot\alpha(v_1,\dots,v_m)$.

1.7. Definition. For every positive integer k we denote by $\mathrm{Der}^k(F)$ the subspace of such elements α in $A^k(F)$ that $\alpha(a,a_1,a_2,\dots,a_k)=a\alpha(a_1,\dots,a_k)+a_1\alpha(a,a_1,a_2,\dots,a_k)$ for every system $\{a,a_1,\dots,a_k\}\subset F$.

Also, we put $\mathrm{Der}^0(F)=F$ and $\mathrm{Der}^*(F)=\overset{\infty}{\underset{k=0}{\oplus}}\mathrm{Der}^k(F)$.

We call elements of the space $\mathrm{Der}^k(F)$ k-derivations of the algebra F, and elements of $\mathrm{Der}^*(F)$ multiderivations.

1.8. It is easy to check that the subspace $\mathrm{Der}^*(F)$ in $A^*(F)$ is closed under the operation of exterior multiplication defined in 1.6 as well as under the bracket defined in 1.2. In other words, $\mathrm{Der}^*(F)$ is an anticommutative algebra and a Lie superalgebra. Moreover, these two structures are connected by the following property: For $\alpha \in \mathrm{Der}^m(F)$, $\beta \in \mathrm{Der}^n(F)$, and $\gamma \in \mathrm{Der}^*(F)$ we have (c) $[\alpha, \beta\gamma] = [\alpha, \beta] \cdot \gamma + (-1)^{mn+n}\beta \cdot [\alpha, \gamma]$.

1.9. For $k = 0, 1, 2, \dots$ let $\wedge^k \mathrm{Der}'(F)$ be the subspace of $\mathrm{Der}^k(F)$ which consists of elements of the form $av_1, \dots, v_k$, where $a \in F$ and $\{v_1, \dots, v_k\} \subset \mathrm{Der}'(F)$. The subalgebra $\wedge^* \mathrm{Der}'(F) = \bigoplus_{k=0}^{\infty} \wedge^k \mathrm{Der}'(F)$ in $\mathrm{Der}^*(F)$ is closed under the bracket [,] which has a more explicit form on the elements of the algebra

$$\wedge^* \mathrm{Der}'(F) : [\alpha_1, \alpha_m, \beta_1, \dots, \beta_n] =$$
$$= \sum_{i,j} (-1)^{m+i+j-1} [\alpha_i, \beta_j] \alpha_1 \cdots \widehat{\alpha}_i \cdots \alpha_m \beta_1 \cdots \widehat{\beta}_j \cdots \beta_n,$$

where $\{\alpha_1, \dots, \alpha_m, \beta_1, \dots, \beta_n\} \subset \mathrm{Der}'(F)$, and $[\alpha_i, \beta_j]$ is the commutator of α_i and β_j.

1.10. A Poisson structure on the commutative algebra F is an involutive element (see 1.4) $P \in \mathrm{Der}^2(F)$. The pair (F, P) is said to be a Poisson algebra.

As mentioned in 1.5, an involutive element $P \in \mathrm{Der}^2(F)$ defines the operator with a vanishing square $\widetilde{P} : \mathrm{Der}^*(F) \to \mathrm{Der}^*(F)$. By virtue of the property (c) of the bracket in $\mathrm{Der}^*(F)$ (see 1.8) it is easy to check that for $\alpha \in \mathrm{Der}^m(F)$ and $\beta \in \mathrm{Der}^n(F)$ we have $\widetilde{P}(\alpha\beta) = \widetilde{P}(\alpha)\beta + (-1)^m \alpha \widetilde{P}(\beta)$. Such an operator is said to be an antiderivation of degree +1.

Therefore, on the space of cohomologies defined by $\widetilde{P}$, we can introduce a structure of anticommutative algebra. This cohomology algebra will be called the cohomology of Poisson structure (F, P). We denote by $H^k(F, P)$ the kth cohomology space, and by $H^*(F, P)$ the comology algebra $\bigoplus_{k=0}^{\infty} H^k(F, P)$.

§ 2. Various Cohomology Algebras of a Manifold with Poisson Structure and Their Interconnections

2.1. As in Section 1, F is a commutative algebra over $\mathbb{R}$ or $\mathbb{C}$.

The space of k-linear antisymmetric homomorphisms of F-modules from $(\mathrm{Der}'(F))^k)$ into F is denoted by $A^k(\mathrm{Der}'(F), F)$, $k = 1, 2, \dots$. It is assumed that $A^0(\mathrm{Der}'(F), F) = F$.

There is a classical operator of derivation on the exterior algebra $A^*(\mathrm{Der}'(F), F) = \bigoplus_{k=0}^{\infty} A^k(\mathrm{Der}'(F), F)$.

2.2. Let P be a Poisson structure on the algebra F. For each $k \in \mathbb{N}$, P defines the homomorphism $P^k : A^k(\mathrm{Der}'(F), F) \to \mathrm{Der}^k(F)$ as follows:

for $a \in A^0(\mathrm{Der}'(F), F) = F$ we put $P^0(a) = a$;

for elements of the form $da \in A'(\mathrm{Der}'(F), F)$, where $a \in F$ and $(da)(X) = X(a)$, we put $p'(da)(b) = P(a, b)$, $b \in F$;

next, for $w \in A^k(\mathrm{Der}'(F), F)$, $k = 1, 2, \dots$, we put $(P^k w)(a_1, \dots, a_k) = (-1)^k w(P'(da_1), \dots, P'(da_k))$ with every system $\{a_1, \dots, a_k\} \subset F$.

2.3. Let us note some interesting properties of P^k, $k = 0, 1, \dots$: The map $P^* = \overset{\infty}{\underset{k=0}{\oplus}} P^* : A^*(\mathrm{Der}'(F), F) \to \mathrm{Der}'(F)$ is a homomorphism of exterior algebras;

Theorem. *The composition map $P' \circ d : F \to \mathrm{Der}'(F)$ is a homomorphism of Lie algebras.*

Proof. We must prove the identity $P'(dP(a, b)) = [P'(da), P'(db)]$ for each $a, b \in F$. By the definitions of P' and $[\ ,\ ]$ we have $P'(dP(a, b))(c) = (P'(da))(P'(db)c) - (P'(db))(P'(da)c) = P(a, P(b, c)) - P(b, P(a, c)))$. Now the identity we want to prove follows from the Jacobi identity for P. $\square$

2.4. Theorem. *The map P^* is a homomorphism from the complex $(A^*(\mathrm{Der}'(F), F), d)$ into the complex $(\mathrm{Der}^*(F), \widetilde{P})$, where d is the classical derivation and $\widetilde{P}$ is defined in* 1.5 *and* 1.11.

Proof. We must prove the identity $P^*(dw) = [P, P^*(w)]$ for every $w \in A^*(\mathrm{Der}'(F), F)$, $n = 0, 1, \dots$. By the definitions we have

$$\begin{aligned}
&P^{n+1}(dw)(a_1, \dots, a_{n+1}) = (-1)^{n+1} dw(P'(da_1), \dots, P'(da_{n+1})) = \\
&= (-1)^{n+1}\Big(\sum_i (-1)^{i-1}(P'(da_i))w\big(P'(da_1), \dots, \widehat{P'(da_i)}, \dots, P'(da_{n+1})\big) + \\
&\quad + \sum_{i<j}(-1)^{i+j} w\big([P'(da_i), P'(da_j)], \dots, \widehat{P'(da_i)}, \dots, \widehat{P'(da_j)}, \dots)\big)\Big) = \\
&= (-1)^{n+1}\Big(\sum_i (-1)^{i-1} P\big(a_i w\big(P'(da_1), \dots, \widehat{P'(da_i)}, \dots, P'(da_{n+1})\big)\big) + \\
&\quad + \sum_{i<j}(-1)^{i+j} w\big([P'(da_i), P'(da_j)], \dots, \widehat{P'(da_i)}, \dots, \widehat{P'(da_j)}, \dots\big)\Big).
\end{aligned}$$

On the other hand,

$$\begin{aligned}
[P, P^n(w)](a_1, \dots, a_{n+1} &= \sum_i (-1)^{i-1} P\big(P^n(w)(a_1, \dots, \widehat{a}_i, \dots, a_{n+1}), a_i\big) + \\
&+ \sum_{i<j}(-1)^{i+j-3}(P^n(w))\big(P(a_i a_j), \dots, \widehat{a}_i, \dots, \widehat{a}_j, \dots\big) =
\end{aligned}$$

$$= (-1)^{n+1}\Big(\sum_i (-1)^{i-1} P\big(a_i w\big(P'(da_1),\ldots,\widehat{P'(da_i)},\ldots,P'(da_{n+1})\big)\big) +$$
$$= \sum_{i<j} (-1)^{i+j} w\big(P'(dP(a_i a_j)),\ldots,\widehat{P'(da_i)},\ldots,\widehat{P'(da_j)},\ldots\big).$$

As in the proof of Theorem 2.3, we obtain $[P'(da_i), P'(da_j)] = P'(dP(a_i, a_j))$, which completes the proof of the theorem. □

As a consequence, P^* defines a homomorphism from the cohomology of the Lie algebra $\mathrm{Der}'(F)$ with coefficients in the algebra F into the cohomology of Poisson structure $H^*(F, P)$.

2.5. Further, we consider the case with $F = C^\infty(M)$, where M is a finite-dimensional C^∞-manifold. Then $\mathrm{Der}^k(F)$ is the space of contravariant antisymmetric tensor fields of degree k on the manifold M and $A^k(\mathrm{Der}'(F), F)$ is the space of differential k-forms on M. These spaces are denoted by $V^k(M)$ and $\Omega^k(M)$, respectively.

The Poisson structure P on M is a contravariant antisymmetric involutive tensor field of degree 2.

P induces a homomorphism $\overline{P} : T^*(M) \to T(M)$ from the cotangent bundle of M into the tangent bundle of M. Let $\beta(P^*_x(\alpha)) = (\alpha \wedge \beta)(P_x)$ for $x \in M$ and $\alpha, \beta \in T^*_x(M)$.

The set of subspaces $\{\mathrm{Image}(\overline{P}_x \subset T_x(M) : x \in M\}$ is an integrable distribution (see [5]). Integral manifolds are called symplectic leaves of the Poisson structure P (see [5], [6]).

Thus we have a foliation $\mathcal{F}_p$ with different-dimensional leaves induced by the Poisson structure P. Now we use the generalization of the cohomology from in 0.6, associated with a submodule on the Lie subalgebra $L \subset V'(M)$.

Let L be the set of vector fields on the manifold M, tangent to the leaves of the foliation $\mathcal{F}_p$.

Since L is a submodule of $V'(M)$ generated by elements of the form $P'(d\varphi)$, where $\varphi \in C^\infty(M)$, it is clear that w is an element of $\Omega^k_L(M)$ if and only if $w(P'(d\varphi_1),\ldots,P'(d\varphi_k)) = 0$ for every system $\{\varphi_1,\ldots,\varphi_k\} \subset C^\infty(M)$; this is the same as $P^k(w) = 0$. So we have $\Omega^*_L(M) = \mathrm{Kernel}(P^*)$.

The consequence of the above result can be formulated as

Theorem. *The cohomology algebra of the complex* $(\Omega^*(M)/\Omega^*_L(M), \widetilde{d})$ (*relative cohomologies*) *is isomorphic to the cohomology algebra of the complex* $(\mathrm{Im}\, P^*, \widetilde{P})$.

2.6. The homomorphism of bundles $\overline{P} : T^*(M) \to T(M)$ induces homomorphisms of the associated bundles $\wedge^k \overline{P} : \wedge^k T^*(M) \to \wedge^k T(M)$, $k = 1, 2, \ldots$. We denote by $V^k(M, P)$ the subspace of $V^k(M)$ consisting of such elements v that $v_x \in \mathrm{Image}\,(\wedge^k \overline{P}^*_x)$ for every $x \in M$. The subalgebra

$V^*(M,P)=\overset{\infty}{\underset{k=0}{\oplus}} V^k(M,P)$ is invariant under the action of th e operator $\widetilde{P}$ (see [7]). Hence we have a complex $(V^*(M,P),\widetilde{P})$ and the corresponding cohomology algebra denoted by $h^*(M,P)$ (see [7]).

Theorem. *The cohomology of the Lie algebra L with coefficients in $C^\infty(M)$ (in other words, the cohomology of the complex $(\Omega^*(M,L),d_L)$ (see 0.6)) is isomorphic to $h^*(M,P)$.*

Proof. We construct a homomorphism $P_L^k:\Omega^k(M,L)\to V^k(M,P)$ for each $k=0,1,\dots,$ analogously to the homomorphisms $P^k:\Omega^k(M)\to V^k(M)$ defined in 2.2. To prove that it is an isomorphism, it is sufficient to show that it is a monomorphism: $P_L^k(w)=0\Rightarrow(P_L^k(w))(\varphi_1,\dots,\varphi_k)=0$ for every $\{\varphi_1,\dots,\varphi_k\}\subset C^\infty(M)\Rightarrow w(P'_L(d\varphi_1),\dots,P'_L(d\varphi_k))=0$. Since L is a module generated by elements of the form $P'_L(d\varphi)$, $\varphi\in C^\infty(M)$, the above identity is equivalent to the identity $w=0$. $\square$

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