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A Note on the Conjecture of Ramirez and Sirvent

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Abstract

We give a proof of a recent conjecture of Ramirez and Sirvent on the generating function of the incomplete Tribonacci numbers.

1 Introduction

In a recent paper of Ramirez and Sirvent [1], the authors have defined the incomplete Tribonacci sequence of numbers and polynomials. They have also studied recurrence relations, some properties of these numbers and polynomials, and the generating function of the incomplete Tribonacci numbers.

Let

$$a_n = \sum_{i=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \sum_{j=0}^{i} i \binom{i}{j} \binom{n-i-j-1}{i}.$$

Ramirez and Sirvent conjectured that the generating function of the sequence $\{a_n\}_{n=0}^{\infty}$ is

$$\sum_{n\geq 0} a_n z^n = \frac{z^3+z^4}{\left(1-z-z^2-z^3\right)^2}.$$

In this short note, we give a proof of this conjecture.

2 A proof of the conjecture

Let $A(z)$ be the generating function of the sequence $\{a_n\}_{n=0}^{\infty}$. Now consider

$$\begin{aligned}
A(z) &= \sum_{0\leq j\leq i} i \binom{i}{j} z^{i+j+1} \sum_{n\geq i} \binom{n-i-j-1}{i} z^{n-i-j-1} \\
&= \sum_{0\leq j\leq i} i \binom{i}{j} z^{i+j+1} \sum_{n\geq i} \binom{n}{i} z^n \\
&= \sum_{0\leq j\leq i} i \binom{i}{j} z^{i+j+1} \frac{z^i}{(1-z)^{i+1}} = \sum_{0\leq j\leq i} i \binom{i}{j} \frac{z^{2i+j+1}}{(1-z)^{i+1}} \\
&= \sum_{i\geq 0} i \frac{z^{2i+1}}{(1-z)^{i+1}} \sum_{0\leq j\leq i} \binom{i}{j} z^j \\
&= \sum_{i\geq 0} i \frac{z^{2i+1}}{(1-z)^{i+1}} (1+z)^i \\
&= \frac{z}{1-z} \sum_{i\geq 0} i \left(\frac{z^2(1+z)}{1-z}\right)^i \\
&= \frac{z}{1-z} \frac{z^2(1-z^2)}{\left(1-z-z^2-z^3\right)^2} \\
&= \frac{z^3+z^4}{\left(1-z-z^2-z^3\right)^2},
\end{aligned}$$

which proves the conjecture.

References

[1] J. L. Ramírez and V. F. Sirvent, Incomplete Tribonacci numbers and polynomials, *J. Integer Sequences*, **17** (2014), Article 14.4.2.

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