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Operators on Differential Forms for Lie Transformation Groups

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Abstract. For any Lie group action $S: G \times P \to P$, we introduce $C^\infty(P)$-linear operators $S^i_\bullet$, that transform n-forms $\omega_n \in \mathcal{A}_n(P,V)$ into $(n-i)$-forms $S^i_\bullet \omega_n \in \mathcal{A}_{n-i}(P, \mathrm{Alt}_i(\mathfrak{g}, V))$. We compute the exterior derivative of these generated forms and their behavior under interior products with vector fields and Lie differentiation. By combination with Lie algebra valued forms $\theta \in \mathcal{A}_1(P,\mathfrak{g})$ and $\phi_p \in \mathcal{A}_p(P,\mathfrak{g})$, we recover V-valued forms $\omega \circledS \theta \in \mathcal{A}_n(P,V)$, resp., $(\chi^s_n \circledS \theta) \bullet \phi_p$ for $\chi^s_n \in \mathcal{A}_n(P, \mathrm{Hom}(\bigotimes^s \mathfrak{g}, V))$ and compute their exterior derivative. The derived formulae play an important role for local evaluations of connections on fiber bundles.

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