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# Transcendance et fonctions modulaires

par François GRAMAIN

ABSTRACT. English title: *Transcendence and modular functions.* After recalling links between elliptic curves and modular functions, we present, with numerous corollaries (in particular the algebraic independence of $\pi$, $e^{\pi}$ and $\Gamma(1/4)$), the theorem obtained by Yuri Nesterenko in 1996: *if $E_2$, $E_4$ and $E_6$ are the three first Eisenstein series and if $q$ satisfies $0 < |q| < 1$ (in both complex and p-adic cases), then the field $\mathbb{Q}(q, E_2(q), E_4(q), E_6(q))$ has transcendence degree at least 3 over $\mathbb{Q}$.* The origin of Nesterenko's work is the proof, in 1995 by a team of Saint-Étienne, of Mahler-Manin conjecture about the modular invariant $J = 1728 \frac{E_4^3}{E_4^3 - E_6^2}$: *if $q \in \mathbb{C}$ or $q \in \mathbb{C}_p$ satisfies $0 < |q| < 1$, then $q$ and $J(q)$ are not both algebraic.* An annex describes the transcendence method of Mahler and why it does not work in this case.

François GRAMAIN
Équipe de Théorie des Nombres
Faculté des Sciences
23, rue du Docteur Paul Michelon
42023 ST ETIENNE CEDEX 2
*E-mail* : gramain@univ-st-etienne.fr