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On a functional-differential equation related to Golomb's self-described sequence

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ABSTRACT. The functional-differential equation $f'(t) = 1/f(f(t))$ is closely related to Golomb's self-described sequence F,

$$\underbrace{1,}_{1,}\ \underbrace{2,2,}_{2,}\ \underbrace{3,3,}_{2,}\ \underbrace{4,4,4}_{3,}\ \underbrace{5,5,5,}_{3,}\ \underbrace{6,6,6,6,}_{4,}\cdots.$$

We describe the increasing solutions of this equation. We show that such a solution must have a nonnegative fixed point, and that for every number $p \geq 0$ there is exactly one increasing solution with p as a fixed point. We also show that in general an initial condition doesn't determine a unique solution: indeed the graphs of two distinct increasing solutions cross each other infinitely many times. In fact we conjecture that the difference of two increasing solutions behaves very similarly as the error term $E(n)$ in the asymptotic expression $F(n) = \phi^{2-\phi}n^{\phi-1} + E(n)$ (where ϕ is the golden number).

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