Journal de Théorie des Nombres
de Bordeaux **12** (2000), 69–79

Topological Properties of Two-Dimensional Number Systems

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Abstract. In the two dimensional real vector space $\mathbb{R}^2$ one can define analogs of the well-known q-adic number systems. In these number systems a matrix M plays the role of the base number q. In the present paper we study the so-called fundamental domain $\mathcal{F}$ of such number systems. This is the set of all elements of $\mathbb{R}^2$ having zero integer part in their "M-adic" representation. It was proved by Kátai and Kőrnyei, that $\mathcal{F}$ is a compact set and certain translates of it form a tiling of the $\mathbb{R}^2$. We construct points, where three different tiles of this tiling coincide. Furthermore, we prove the connectedness of $\mathcal{F}$ and give a result on the structure of its inner points.

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Manuscrit reçu le 15 janvier 1999.