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Réalisation de formes $\mathbb{Z}$-bilinéaires symétriques comme formes trace hermitiennes amplifiées

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ABSTRACT. In this paper, we show by an explicit method that every non degenerate symmetric $\mathbb{Z}$-bilinear form of even rank, which is not $\mathbb{Q}$-isomorphic to the hyperbolic plane, can be realized as a hermitian scaled trace form of some algebra $\mathbb{Z}[\alpha]$, where α is an algebraic integer. More precisely, we show that for every symmetric matrix $S \in M_{2n}(\mathbb{Z})$, with $\det S \neq 0$ (and $\det S \not\equiv -1 \pmod{\mathbb{Q}^{*2}}$ if $n = 1$), there exist an algebraic integer α, a $\mathbb{Q}$-linear involution σ of $\mathbb{Q}(\alpha)$, a σ-symmetric element $\lambda \in \mathbb{Q}(\alpha)$ and a $\mathbb{Z}$-basis $v_1, \cdots, v_{2n}$ of some ideal of $\mathbb{Z}[\alpha]$ such that $S = (\mathrm{Tr}_{\mathbb{Q}(\alpha)/\mathbb{Q}}(\lambda v_i v_j^\sigma))$.

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