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On integral representations by totally positive ternary quadratic forms

par Elise BJÖRKHOLDT

Abstract. Let K be a totally real algebraic number field whose ring of integers R is a principal ideal domain. Let $f(x_1, x_2, x_3)$ be a totally definite ternary quadratic form with coefficients in R. We shall study representations of totally positive elements $N \in R$ by f. We prove a quantitative formula relating the number of representations of N by different classes in the genus of f to the class number of $R[\sqrt{-c_f N}]$, where $c_f \in R$ is a constant depending only on f. We give an algebraic proof of a classical result of H. Maass on representations by sums of three squares over the integers in $\mathbb{Q}(\sqrt{5})$ and obtain an explicit dependence between the number of representations and the class number of the corresponding biquadratic field. We also give similar formulae for some quadratic forms arising from maximal quaternion orders, with class number one, over the integers in real quadratic number fields.

Elise Björkholdt
Department of Mathematics,
Chalmers University of Technology
and Göteborg University,
S-412 96 Göteborg, SWEDEN
E-mail : elise@math.chalmers.se

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