Journal de Théorie des Nombres
de Bordeaux **14** (2002), 113–149

On Galois structure of the integers in cyclic extensions of local number fields

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Dedicated to Professor Manohar L. Madan

Résumé. Soit p un nombre premier, K une extension finie du corps des nombres p-adiques, et L/K une extension cyclique ramifiée de degré p^n. On suppose que le premier nombre de ramification de L/K satisfait $b_1 > 1/2 \cdot pe_0/(p-1)$ où e_0 est l'indice de ramification absolu de K. Nous déterminons explicitement la structure de l'anneau des entiers de L comme $\mathbb{Z}_p[G]$-module, où $\mathbb{Z}_p$ désigne l'anneau des entiers p-adiques et $G = \mathrm{Gal}(L/K)$ le groupe de Galois de L.

Abstract. Let p be a rational prime, K be a finite extension of the field of p-adic numbers, and let L/K be a totally ramified cyclic extension of degree p^n. Restrict the first ramification number of L/K to about half of its possible values, $b_1 > 1/2 \cdot pe_0/(p-1)$ where e_0 denotes the absolute ramification index of K. Under this loose condition, we explicitly determine the $\mathbb{Z}_p[G]$-module structure of the ring of integers of L, where $\mathbb{Z}_p$ denotes the p-adic integers and G denotes the Galois group $\mathrm{Gal}(L/K)$. In the process of determining this structure, we study various restrictions on the ramification filtration and examine the trace map relationships that result. Two of these restrictions are generalizations of *almost maximal ramification*. Our method for determining this structure is constructive (also inductive). We exhibit generators for the ring of integers of L over the group ring, $\mathbb{Z}_p[G]$ (actually over $\mathfrak{O}_T[G]$ where $\mathfrak{O}_T$ is the ring of integers in the maximal unramified subfield of K). They are determined in an essential way by their valuation. Then we describe their relations.

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Manuscrit reçu le 2 février 2001.
This research was partially supported through UCR grant no. 99-28, University of Nebraska at Omaha.