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Polynomial growth of sumsets in abelian semigroups

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To Michel Mendès France

Résumé. Soit S un semi-groupe abélien et A un sous-ensemble fini de S. On désigne par hA l'ensemble de toutes les sommes de h éléments de A, et par $|hA|$ son cardinal. On montre, par des arguments élémentaires de comptage de points dans les réseaux, qu'il existe un polynôme $p(t)$ tel que pour tout entier h assez grand $|hA| = p(h)$. Plus généralement, on étend ce résultat aux ensembles $h_1A_1+\cdots+h_rA_r$ en obtenant la croissance polynomiale du cardinal en termes des variables $h_1, h_2, \ldots, h_r$.

Abstract. Let S be an abelian semigroup, and A a finite subset of S. The sumset hA consists of all sums of h elements of A, with repetitions allowed. Let $|hA|$ denote the cardinality of hA. Elementary lattice point arguments are used to prove that an arbitrary abelian semigroup has polynomial growth, that is, there exists a polynomial $p(t)$ such that $|hA| = p(h)$ for all sufficiently large h. Lattice point counting is also used to prove that sumsets of the form $h_1A_1 + \cdots + h_rA_r$ have multivariate polynomial growth.

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