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A NOTE ON GENERALIZED GORENSTEIN DIMENSION

(submitted by M. M. Arslanov)

ABSTRACT. We prove that two categories \mathcal{G}_ω and \mathcal{X}_ω , introduced for the faithfully balanced selforthogonal module ω by Auslander and Reiten in [2] and [3] respectively, coincide with each other. As an application we give a generalization of a main theorem in [6].

1. INTRODUCTION

Throughout this note, we assume that R (S respectively) is a left (right respectively) noetherian ring and modules are finitely generated left modules. By a subcategory we mean a full subcategory closed under isomorphisms. We fix ω a faithfully balanced selforthogonal R -module with $S = \text{End}_R \omega$ (i.e. a generalized tilting module in sense of [8]), which is defined to satisfy the conditions (a) $R \simeq \text{End}_S \omega$ and, (b) $\text{Ext}_R^i(\omega, \omega) = 0 = \text{Ext}_S^i(\omega, \omega)$ for all $i \geq 1$.

The notion of the Gorenstein dimension was introduced in [1] for a two-sided noetherian ring R . An R -module M has Gorenstein dimension zero, written $\text{G-dim} M = 0$, if M is a reflexive R -module and $\text{Ext}_R^i(M, R) = 0 = \text{Ext}_R^i(M^*, R)$ for all $i \geq 1$, where $M^* = \text{Hom}_R(M, R)$. Further, $\text{G-dim} M = n$ if n is the minimal integer such that there is an exact sequence $0 \rightarrow L_n \rightarrow \cdots \rightarrow L_0 \rightarrow M \rightarrow 0$ with $\text{G-dim} L_i = 0$ for all $0 \leq i \leq n$. Otherwise, $\text{G-dim} M = \infty$.

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In [2], the authors studied a generalization of this dimension. An R -module M is said to be of generalized Gorenstein dimension zero, written $\text{G-dim}_\omega M = 0$, if $\text{Ext}_R^i(M, \omega) = 0 = \text{Ext}_S^i(\text{Hom}_R(M, \omega), \omega)$ for all $i \geq 1$ and the canonical map $\sigma_M : M \rightarrow \text{Hom}_S(\text{Hom}_R(M, \omega), \omega)$ is an isomorphism. $\text{G-dim}_\omega M = n$ if n is the minimal integer such that there is an exact sequence $0 \rightarrow L_n \rightarrow \cdots \rightarrow L_0 \rightarrow M \rightarrow 0$ with $\text{G-dim}_\omega L_i = 0$ for all $0 \leq i \leq n$, otherwise, $\text{G-dim}_\omega M = \infty$. We denoted by \mathcal{G}_ω the subcategory of R -modules of generalized Gorenstein dimension zero.

It was also introduced in [3] a subcategory, denoted by \mathcal{X}_ω , whose objects are R -modules M such that there is an exact sequence $0 \rightarrow M \rightarrow \omega_0 \xrightarrow{f_0} \omega_1 \xrightarrow{f_1} \cdots$ with $\omega_i \in \text{add}_R \omega$, i.e. the subcategory of the direct summands of finite direct sum of ω , with $\text{Im} f_j \in {}^\perp \omega = \{X \in R\text{-mod} \mid \text{Ext}_R^i(X, \omega) = 0 \text{ for all } i \geq 1\}$ for all $j \geq 0$.

One purpose of this note is to study the relation between the above-referenced two subcategories. It is well known that $\mathcal{G}_\omega \subseteq \mathcal{X}_\omega$ and $\mathcal{G}_\omega = \mathcal{X}_\omega$ when ω is a cotilting bimodule (see [2] for the case of artin algebras and [6] for the case of two-sided noetherian ring). In this note, we will show that $\mathcal{G}_\omega = \mathcal{X}_\omega$ always holds for the faithfully balanced selforthogonal module ω . Note that the cotilting bimodule (in artin algebras [3] and in [6]) is faithfully balanced selforthogonal, but conversely the faithfully balanced selforthogonal module need not be cotilting (see for instance [9, Example 3.1]). As the Gorenstein dimension and its generalizations were recently studied by many mathematicians (e.g. [4, 5, 7] etc.), this result would be helpful for us to understand the (generalized) Gorenstein dimension more precisely.

We in turn compare the generalized Gorenstein dimension with the left orthogonal dimension related to ω defined in [6]. Accordantly, an R -module M has left orthogonal dimension related to ω equal to n , written ${}^\perp \omega\text{-dim} M = n$, if n is the minimal integer such that there is an exact sequence $0 \rightarrow K_n \rightarrow \cdots \rightarrow K_0 \rightarrow M \rightarrow 0$ with $K_i \in {}^\perp \omega$ for all $0 \leq i \leq n$, otherwise, ${}^\perp \omega\text{-dim} M = \infty$. We show that ${}^\perp \omega\text{-dim} M \leq \text{G-dim}_\omega M$ in general and they agree when the latter is finite. Consequently, we get a method to compute the generalized Gorenstein dimension when it is known to be finite. We also study the subcategory of modules of finite generalized Gorenstein dimension, denoted by $\widehat{\mathcal{G}}_\omega$, and some known results were extended.

We denote by $R\text{-mod}$ ($\text{mod-}S$ respectively) the category of all finitely generated left R -modules (right S -modules respectively). Let \mathcal{C} be a subcategory of $A\text{-mod}$, we denote by $\widehat{\mathcal{C}}$ the category of all modules M such that there is an exact sequence $0 \rightarrow C_m \rightarrow \cdots \rightarrow C_0 \rightarrow M \rightarrow 0$ for

some integer m with each $C_i \in \mathcal{C}$. Assume $\mathcal{C} \supset \mathcal{D}$ are two subcategories of $R\text{-mod}$. Let $C \in \mathcal{C}$ and $D \in \text{add}_R \mathcal{D}$. A homomorphism $D \rightarrow C$ is said to be a right \mathcal{D} -approximation of C if $\text{Hom}_R(X, D) \rightarrow \text{Hom}_R(X, C)$ is an epimorphism for any $X \in \text{add}_R \mathcal{D}$. \mathcal{D} is said to be a contravariantly finite subcategory of \mathcal{C} (or \mathcal{D} is contravariantly finite in \mathcal{C}) if every $C \in \mathcal{C}$ has a right \mathcal{D} -approximation. Dually, A homomorphism $C \rightarrow D$ is said to be a left \mathcal{D} -approximation of C if $\text{Hom}_R(D, X) \rightarrow \text{Hom}_R(C, X)$ is an epimorphism for any $X \in \text{add}_R \mathcal{D}$. \mathcal{D} is said to be a covariantly finite subcategory of \mathcal{C} (or \mathcal{D} is covariantly finite in \mathcal{C}) if every $C \in \mathcal{C}$ has a left \mathcal{D} -approximation. \mathcal{D} is said to be a functionally finite subcategory of \mathcal{C} (or \mathcal{D} is functionally finite in \mathcal{C}) if \mathcal{D} is both a contravariantly finite subcategory and a covariantly finite subcategory of \mathcal{C} .

Assume that \mathcal{D} is contravariantly finite subcategory in \mathcal{C} and \mathcal{D}' is covariantly finite subcategory in \mathcal{C} . If every kernel of right \mathcal{D} -approximations is in \mathcal{D}' , then \mathcal{D}' is called the associated (with \mathcal{D}) covariantly finite subcategory. Similarly, if every cokernel of left \mathcal{D}' -approximations is in \mathcal{D} , then \mathcal{D} is called the associated (with \mathcal{D}') contravariantly finite subcategory.

For $A \in R\text{-mod}$ ($\text{mod-}S$ respectively), we put $A^\omega = \text{Hom}_R(A, \omega)$ ($\text{Hom}_S(A, \omega)$ respectively). An R -module M is said to be ω -reflexive if the natural homomorphism $\sigma_M : M \rightarrow M^{\omega\omega}$ is an isomorphism.

2. MAIN RESULTS

Theorem 2.1. $\mathcal{G}_\omega = \mathcal{X}_\omega$, for the faithfully balanced selforthogonal R -module ω .

Proof. It's sufficient to show $\mathcal{X}_\omega \subseteq \mathcal{G}_\omega$ by [2, Proposition 4.3]. Obviously we have $\mathcal{X}_\omega \subseteq {}^\perp \omega$. Now let $M \in \mathcal{X}_\omega$. By the definition we see that there exists an exact sequence $0 \rightarrow M \rightarrow \omega_0 \rightarrow N \rightarrow 0$ with $\omega_0 \in \text{add}_R \omega$ and $N \in \mathcal{X}_\omega$. Note that $N \in {}^\perp \omega$, so that the induced sequence $0 \rightarrow N^\omega \rightarrow \omega_0^\omega \rightarrow M^\omega \rightarrow 0$ is exact. Hence we have the following exact commutative diagram, where $\sigma_N, \sigma_{\omega_0}, \sigma_M$ are the natural homomorphisms.

$$\begin{array}{ccccccc} 0 & \longrightarrow & M & \longrightarrow & \omega_0 & \longrightarrow & N \longrightarrow 0 \\ & & \sigma_M \downarrow & & \sigma_{\omega_0} \downarrow & & \sigma_N \downarrow \\ 0 & \longrightarrow & M^{\omega\omega} & \longrightarrow & \omega_0^{\omega\omega} & \longrightarrow & N^{\omega\omega} \longrightarrow \text{Ext}_S^1(M^\omega, \omega) \longrightarrow 0 \end{array}$$

It is easy to see that σ_{ω_0} is a natural isomorphism, so we have that σ_M is a monomorphism. Note that N has properties same as M , so

σ_N is a monomorphism, too. Therefore σ_M must be an isomorphism. In the same way we obtain that σ_N is an isomorphism. It follows that $\text{Ext}_S^1(M^\omega, \omega) = 0$. Since $\text{Ext}_S^{i+1}(M^\omega, \omega) \simeq \text{Ext}_S^i(N^\omega, \omega)$ for $i \geq 1$, we see that $\text{Ext}_S^i(M^\omega, \omega) = 0$ for all $i \geq 1$, by repeating the process. Hence M is an R -module of generalized Gorenstein dimension zero and it follows $\mathcal{X}_\omega \subseteq \mathcal{G}_\omega$. \square

Corollary 2.2. *\mathcal{G}_ω is closed under extensions, kernels of epimorphisms and direct summands.*

Proof. By Theorem 2.1 and [3, Proposition 5.1] (the proof can be transferred directly to our settings). \square

The following proposition compares the generalized Gorenstein dimension with the left orthogonal dimension relative to ω .

Proposition 2.3. *Let $M \in R\text{-mod}$. Then ${}^\perp\omega - \dim M \leq G\text{-dim}_\omega M$ and they agree when the latter is finite.*

Proof. Since $\mathcal{G}_\omega \subseteq {}^\perp\omega$, we see that ${}^\perp\omega - \dim M \leq G\text{-dim}_\omega M$ by the definitions. Now let $G\text{-dim}_\omega M = t < \infty$ and ${}^\perp\omega - \dim M = s$. By [6, Lemma 7], $s = \sup\{k \mid \text{Ext}_R^k(M, \omega) \neq 0\}$. Hence $\text{Ext}_R^i(M, \omega) = 0$ for all $i > s$. Consider the sequence $0 \rightarrow G_t \xrightarrow{f} G_{t-1} \rightarrow \cdots \rightarrow G_0 \rightarrow M \rightarrow 0$ with $G_i \in \mathcal{G}_\omega$ for $0 \leq i \leq t$. Let $N = \text{Coker } f$. If $t > s$ strictly, then we have that $\text{Ext}_R^i(N, \omega) \simeq \text{Ext}_R^{t-1+i}(M, \omega) = 0$ for all $i \geq 1$. Hence the sequence $0 \rightarrow N^\omega \rightarrow G_{t-1}^\omega \rightarrow G_t^\omega \rightarrow 0$ is exact. It follows that $0 \rightarrow G_t^{\omega\omega} \rightarrow G_{t-1}^{\omega\omega} \rightarrow N^{\omega\omega} \rightarrow 0$ is also exact and $\text{Ext}_S^i(N^\omega, \omega) = 0$ for $i \geq 1$. Now it is easy to see that N is ω -reflexive. Hence $N \in \mathcal{G}_\omega$. Therefore we have $G\text{-dim}_\omega M \leq t - 1$, which is a contradiction. \square

Consequently, we get a method to compute the generalized Gorenstein dimension when it is known to be finite.

Corollary 2.4. *Assume that $G - \dim_\omega M < \infty$. Then $G - \dim_\omega M = \sup\{k \mid \text{Ext}_R^k(M, \omega) \neq 0\}$.*

Proof. By Proposition 2.3 and [6, Lemma 7]. \square

The following lemma generalizes [1, Theorem 3.13].

Lemma 2.5. *For an integer $k \geq 0$ and $M \in R\text{-mod}$, the following are equivalent:*

- (1) $G\text{-dim}_\omega \Omega_{\mathcal{G}_\omega}^k(M) = 0$, where $\Omega_{\mathcal{G}_\omega}^k(M)$ denote k -syzygies of the \mathcal{G}_ω -resolution of M .
- (2) $G\text{-dim}_\omega \Omega^k(M) = 0$.
- (3) $G\text{-dim}_\omega M \leq k$.

Proof. (1) \Rightarrow (2). It follows from the fact that \mathcal{G}_ω contains all finitely generated projective R -modules.

(2) \Rightarrow (3). By the definition.

(3) \Rightarrow (1). By Corollary 2.2 and the fact that \mathcal{G}_ω contains all finitely generated projective R -modules, we see that [1, Lemma 3.12] works. It follows that (1) holds. \square

Corollary 2.6. *Let $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ be exact in $R\text{-mod}$. If two of A, B, C are in $\widehat{\mathcal{G}}_\omega$ then the third is also in $\widehat{\mathcal{G}}_\omega$. Moreover,*

- (1) *For any $n > G\text{-dim}_\omega C$, $G\text{-dim}_\omega A \leq n$ if and only if $G\text{-dim}_\omega B \leq n$.*
- (2) *If $G\text{-dim}_\omega B = 0 \neq G\text{-dim}_\omega C$, then $G\text{-dim}_\omega A = G\text{-dim}_\omega C - 1$.*

Proof. It's easy to see that, for any integer $t \geq 1$, there is the following exact commutative diagram, where P_i, Q_i , $0 \leq i \leq t-1$, are finitely generated projective R -modules and $\Omega^t(A)$, $\Omega^t(B)$, $\Omega^t(C)$ are some t -syzygies of A, B, C respectively.

$$\begin{array}{ccccccc}
 & & 0 & & 0 & & 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 0 & \longrightarrow & \Omega^t(A) & \longrightarrow & \Omega^t(B) & \longrightarrow & \Omega^t(C) \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 0 & \longrightarrow & P_{t-1} & \longrightarrow & P_{t-1} \oplus Q_{t-1} & \longrightarrow & Q_{t-1} \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 & & \vdots & & \vdots & & \vdots \\
 & & \downarrow & & \downarrow & & \downarrow \\
 0 & \longrightarrow & P_0 & \longrightarrow & P_0 \oplus Q_0 & \longrightarrow & Q_0 \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 & & 0 & & 0 & & 0
 \end{array}$$

If two of $\Omega^t(A)$, $\Omega^t(B)$, $\Omega^t(C)$ are modules of generalized Gorenstein dimension zero, then the generalized Gorenstein dimension of the third is clearly not more than one. It follows from Lemma 2.5 that, if two of A, B, C in the exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ have finite generalized Gorenstein dimension, then the third also has finite generalized Gorenstein dimension.

For any $n > \text{G-dim}_\omega C$ we easily see that $\Omega^n(C) \in \mathcal{G}_\omega$. By Corollary 2.2, \mathcal{G}_ω is closed under extensions and kernels of epimorphisms. Hence $\Omega^n(A) \in \mathcal{G}_\omega$ if and only if $\Omega^n(B) \in \mathcal{G}_\omega$. It follows again by Lemma 2.5 that $\text{G-dim}_\omega A \leq n$ if and only if $\text{G-dim}_\omega B \leq n$.

Assume that $\text{G-dim}_\omega B = 0$. If $\text{G-dim}_\omega C = 1$, then by Lemma 2.5 we have $\text{G-dim}_\omega A = 0$. If $\text{G-dim}_\omega C > 1$, then $\sup\{k \mid \text{Ext}_R^k(C, \omega) \neq 0\} = \sup\{k \mid \text{Ext}_R^k(A, \omega) \neq 0\} + 1$ since $\text{Ext}_R^{i+1}(C, \omega) \simeq \text{Ext}_R^i(A, \omega)$ for all $i \geq 1$. By Proposition 2.3 we see that $\text{G-dim}_\omega A = \text{G-dim}_\omega C - 1$. \square

The following lemma is obvious (cf. [6, Lemma 1]).

Lemma 2.7. *For a faithfully balanced selforthogonal R -module ω , it holds that $\text{Ext}_R^i({}^\perp\omega, \widehat{\text{add}_R\omega}) = 0$ for all $i \geq 1$.*

Lemma 2.8. *For any $N \in \widehat{\mathcal{G}_\omega}$, there exist two exact sequences $0 \rightarrow L \rightarrow G \rightarrow N \rightarrow 0$ and $0 \rightarrow N \rightarrow M \rightarrow G' \rightarrow 0$ with $G, G' \in \mathcal{G}_\omega$ and $L, M \in \widehat{\text{add}_R\omega}$ such that $G \rightarrow N$ is a right \mathcal{G}_ω -approximation of N and $N \rightarrow M$ is a left $\widehat{\text{add}_R\omega}$ -approximation of N .*

Proof. For any $N \in \widehat{\mathcal{G}_\omega}$, let $t(N) = \text{G-dim}_\omega N < \infty$. We will proceed by induction on $t(N)$.

If $t(N) = 0$ i.e., $N \in \mathcal{G}_\omega$, then let $L = 0$ and $G = N$, and the first desired exact sequence follows. By Theorem 1, we see that there is an exact sequence $0 \rightarrow N \rightarrow \omega_0 \rightarrow G' \rightarrow 0$ such that $\omega_0 \in \text{add}_R\omega$ and $G' \in \mathcal{G}_\omega$. It's easy to see that this is just the second desired exact sequence by Lemma 2.7.

Let $t(N) \geq 1$. Consider the exact sequence $0 \rightarrow M \rightarrow P_0 \rightarrow N \rightarrow 0$ with P_0 finitely generated projective. By Corollary 2.6, $t(M) = t(N) - 1$ since P_0 is clearly in \mathcal{G}_ω . Therefore, there is an exact sequence $0 \rightarrow M \rightarrow B \rightarrow C \rightarrow 0$ with $B \in \widehat{\text{add}_R\omega}$ and $C \in \mathcal{G}_\omega$, by the induction assumptions. Now consider the following pushout diagram.

$$\begin{array}{ccccccc}
 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \\
 0 & \longrightarrow & M & \longrightarrow & P_0 & \longrightarrow & N \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \parallel \\
 0 & \longrightarrow & B & \longrightarrow & D & \longrightarrow & N \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \\
 & & C & \xlongequal{\quad} & C & & \\
 & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & &
 \end{array}$$

It follows from the middle column that $D \in \mathcal{G}_\omega$, since \mathcal{G}_ω is closed under extensions. By Lemma 2.7 we see that the middle row is just the desired first exact sequence. Moreover, note that there is an exact sequence $0 \rightarrow D \rightarrow \omega'_0 \rightarrow A \rightarrow 0$ such that $\omega'_0 \in \text{add}_R \omega$ and $A \in \mathcal{G}_\omega$, so we have the following exact commutative diagram.

$$\begin{array}{ccccccc}
 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \\
 0 & \longrightarrow & B & \longrightarrow & D & \longrightarrow & N \longrightarrow 0 \\
 & & \parallel & & \downarrow & & \downarrow \\
 0 & \longrightarrow & B & \longrightarrow & \omega'_0 & \longrightarrow & E \longrightarrow 0 \\
 & & & & \downarrow & & \downarrow \\
 & & & & A & \xlongequal{\quad} & A \\
 & & & & \downarrow & & \downarrow \\
 & & & & 0 & & 0
 \end{array}$$

It follows from the middle row that $E \in \widehat{\text{add}_R \omega}$, since $B \in \widehat{\text{add}_R \omega}$. By Lemma 2.7 we see that the middle column is just the desired second exact sequence. \square

By the above lemma and the definition of homologically finite subcategories, we have the following proposition.

Proposition 2.9. *\mathcal{G}_ω is a contravariantly finite subcategory of $\widehat{\mathcal{G}_\omega}$ and $\widehat{\text{add}_R \omega}$ is the associated covariantly finite subcategory of $\widehat{\mathcal{G}_\omega}$.*

Lemma 2.10. *If for any $M \in \widehat{\mathcal{G}_\omega}$, M^ω (as a right S -module) has finite generalized Gorenstein dimension, then \mathcal{G}_ω is covariantly finite in $\widehat{\mathcal{G}_\omega}$.*

Proof. As in Lemma 2.8, we obtain that, for any $M^\omega \in \text{mod } -S$, there exists an exact sequence $0 \rightarrow A' \rightarrow X \xrightarrow{f} M^\omega \rightarrow 0$ such that $f : X \rightarrow M^\omega$ is a right \mathcal{G}_ω -approximation of M^ω and $A' \in \widehat{\text{add}_S \omega}$. Let $h = f^\omega \circ \sigma_M$ where $\sigma_M : M \rightarrow M^{\omega\omega}$ is the canonical homomorphism, we will show that $h : M \rightarrow X^\omega$ is a left \mathcal{G}_ω -approximation of M (note that X^ω has generalized Gorenstein dimension zero by the definition).

Let $g : M \rightarrow G$ be any homomorphism of R -modules with $G \in \mathcal{G}_\omega$. Consider the following diagram.

$$\begin{array}{ccccc} M & \xrightarrow{\sigma_M} & M^{\omega\omega} & \xrightarrow{f^\omega} & X^\omega \\ \downarrow g & & \downarrow g^{\omega\omega} & \swarrow \phi^\omega & \\ G & \xrightarrow{\sigma_G} & G^{\omega\omega} & & \end{array}$$

By Lemma 2.7, we have $\text{Ext}_S^1(G^\omega, A') = 0$. It follows that $\text{Hom}_S(G^\omega, X) \rightarrow \text{Hom}_S(G^\omega, M^\omega)$ is an epimorphism. Hence there is a homomorphism of right S -modules $\phi : G^\omega \rightarrow X$ such that $g^\omega = f \circ \phi$. Consequently we have $g^{\omega\omega} = \phi^\omega \circ f^\omega$. Note that σ_G is an isomorphism by the definition and that $\sigma_G \circ g = g^{\omega\omega} \circ \sigma_M$, so we have $g = \sigma_G^{-1} \circ g^{\omega\omega} \circ \sigma_M = \sigma_G^{-1} \circ \phi^\omega \circ f^\omega \circ \sigma_M = \sigma_G^{-1} \circ \phi^\omega \circ h$. It follows that $\text{Hom}_R(X^\omega, G) \rightarrow \text{Hom}_R(M, G)$ is an epimorphism. Then, by the definition $h : M \rightarrow G^\omega$ is a left \mathcal{G}_ω -approximation of M .

□

Combining Proposition 2.9 and Lemma 2.10, we obtain the following theorem.

Proposition 2.11. *Assume the same assumptions as in Lemma 2.10, then \mathcal{G}_ω is functionally finite in $\widehat{\mathcal{G}_\omega}$.*

An R -module ω is said to be a cotilting bimodule if it is faithfully balanced selforthogonal and $\text{id}_R \omega = \text{id}_S \omega < \infty$. It is well known that $\widehat{\mathcal{G}_\omega} = R - \text{mod}$ if ω is a cotilting bimodule (see [2] and [6]). Hence we have the following corollary as a special case of theorem 2.11.

Corollary 2.12. [6, Theorem 1] *If ω is a cotilting bimodule, then \mathcal{G}_ω is functionally finite in $R - \text{mod}$.*

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