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GROUPOID ENRICHED CATEGORIES AND NATURAL SYSTEMS

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ABSTRACT. We generalize the Baues-Jibladze descent theorem to a large class of groupoid enriched categories.

1. Introduction

A natural system D on a category $\boldsymbol{C}$ is a covariant functor on the category of factorizations $\mathbf{F}\boldsymbol{C}$ of $\boldsymbol{C}$ (also called twisted arrow category of $\boldsymbol{C}$, see [Mac Lane 1971]). Cohomology of $\boldsymbol{C}$ with coefficients in a natural system was first defined in [Baues & Wirsching 1985] and plays important rôle in many areas of algebra and topology [Baues 1989, Baues 2003, Jibladze & Pirashvili 1991, Jibladze & Pirashvili 2005, Pirashvili 1990]. One of the main points for the applications is the fact that the third cohomology group $H^3(\boldsymbol{C}; D)$ classifies the so called linear track extensions of $\boldsymbol{C}$ by D [Pirashvili 1988, Pirashvili 1990]. Recently in [Baues & Jibladze 2002] it was proved that linear track extensions are essentially the same as groupoid enriched categories such that automorphism groups of all 1-arrows are abelian (=abelian track categories, see below). The proof of this important result relies on the fact that in any abelian track category $\mathscr{T}$, automorphism groups $\mathrm{Aut}_{\mathscr{T}}(f)$ of 1-arrows can be "descended" to the homotopy category $\mathscr{T}_{\simeq}$, i. e. they only depend on the isomorphism class of f in a nice way – see Theorem 2.4 in [Baues & Jibladze 2002].

The aim of this work is to generalize this descent result for a large class of non-abelian natural systems equipped with certain type of descent data.

2. Preliminaries

A groupoid is called *abelian* if the automorphism group of each object is an abelian group. We will use the following notation for 2-categories. Composition of 1-arrows will be denoted by juxtaposition; for 2-arrows we will use additive notation, so composition is $+$ and identity 2-arrows are denoted by 0. The hom-category for objects A, B of a 2-category will be denoted by $[\![A, B]\!]$.

There are several categories associated with a 2-category $\mathscr{T}$. The category $\mathscr{T}_0$ has the same objects as $\mathscr{T}$, while morphisms in $\mathscr{T}_0$ are 1-arrows of $\mathscr{T}$. The category $\mathscr{T}_1$ has the same objects as $\mathscr{T}_0$. The morphisms $A \to B$ in $\mathscr{T}_1$ are 2-arrows $\alpha : f \Rightarrow f_1$ where

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$f, f_1 : A \to B$ are 1-arrows in $\mathscr{T}$. The composition in $\mathscr{T}_1$ is given by $(\beta : x \Rightarrow x_1)(\alpha : f \Rightarrow f_1) := (\beta\alpha : xf \Rightarrow x_1 f_1)$, where

$$\beta\alpha = \beta f_1 + x\alpha = x_1\alpha + \beta f.$$

One furthermore has the source and target functors

$$\mathscr{T}_1 \underset{t}{\overset{s}{\rightrightarrows}} \mathscr{T}_0 \,,$$

where $s(\alpha : f \Rightarrow f_1) = f$ and $t(\alpha : f \Rightarrow f_1) = f_1$, the "identity" functor $i : \mathscr{T}_0 \to \mathscr{T}_1$ assigning to an 1-arrow f the triple $0_f : f \Rightarrow f$. Moreover, consider the pullback diagram

$$(*) \qquad \begin{tikzcd} \mathscr{T}_1 \times_{\mathscr{T}_0} \mathscr{T}_1 \arrow[r, "p_2"] \arrow[d, "p_1"'] & \mathscr{T}_1 \arrow[d, "t"] \\ \mathscr{T}_1 \arrow[r, "s"] & \mathscr{T}_0 \end{tikzcd} ;$$

there is also the "composition" functor $m : \mathscr{T}_1 \times_{\mathscr{T}_0} \mathscr{T}_1 \to \mathscr{T}_1$ sending $(\alpha : f \Rightarrow f_1, \alpha' : f_2 \Rightarrow f)$ to $\alpha + \alpha' : f_2 \Rightarrow f_1$. Note that these functors satisfy the identities $sp_1 = tp_2$, $sm = sp_2$, $tm = tp_1$ and $si = ti = \mathrm{id}_{\mathscr{T}_0}$. Sometimes we will also simply write $\mathscr{T}_1 \rightrightarrows \mathscr{T}_0$ to indicate a 2-category $\mathscr{T}$.

A *track category* $\mathscr{T}$ is a *category enriched in groupoids*, i. e. is the same as a 2-category all of whose 2-arrows are invertible. If the groupoids $[\![A, B]\!]$ are abelian for all $A, B \in \mathrm{Ob}\mathscr{T}$, then $\mathscr{T}$ is called an *abelian* track category. For track categories we might occasionally talk about *maps* instead of 1-arrows and *homotopies* or *tracks* instead of 2-arrows. If there is a homotopy $\alpha : f \Rightarrow g$ between maps $f, g \in \mathrm{Ob}([\![A, B]\!])$, we will say that f and g are homotopic and write $f \simeq g$. Since the homotopy relation is a natural equivalence relation on morphisms of $\mathscr{T}_0$, it determines the *homotopy category* $\mathscr{T}_\simeq = \mathscr{T}_0/\simeq$. Objects of $\mathscr{T}_\simeq$ are once again objects in $\mathrm{Ob}(\mathscr{T})$, while morphisms of $\mathscr{T}_\simeq$ are homotopy classes of morphisms in $\mathscr{T}_0$. For objects A and B we let $[A, B]$ denote the set of morphisms from A to B in the category $\mathscr{T}_\simeq$. Thus

$$[A, B] = [\![A, B]\!] / \simeq .$$

Usually we let $q : \mathscr{T}_0 \to \mathscr{T}_\simeq$ denote the quotient functor. Sometimes for a 1-arrow f in $\mathscr{T}$ we will denote $q(f)$ by $[f]$. A map $f : A \to B$ is a *homotopy equivalence* if there exists a map $g : B \to A$ and tracks $fg \simeq 1$ and $gf \simeq 1$. This is the case if and only if $q(f)$ is an isomorphism in the homotopy category $\mathscr{T}_\simeq$. In this case A and B are called *homotopy equivalent* objects.

A *track functor* $F : \mathscr{T} \to \mathscr{T}'$ between track categories is by definition a groupoid enriched functor. Let $\boldsymbol{C}$ be a category. Then the category $\mathbf{F}\boldsymbol{C}$ of factorizations in $\boldsymbol{C}$ is defined as follows. Objects of $\mathbf{F}\boldsymbol{C}$ are morphisms $f : A \to B$ in $\boldsymbol{C}$ and morphisms

$(a, b) : f \to g$ in $\mathbf{F}\boldsymbol{C}$ are commutative diagrams

$$\begin{tikzcd} A \arrow[d, "f"'] & A' \arrow[l, "a"'] \arrow[d, "g"] \\ B \arrow[r, "b"'] & B' \end{tikzcd}$$

in the category $\boldsymbol{C}$. A *natural system* on $\boldsymbol{C}$ with values in a category $\mathscr{C}$ is a covariant functor $D : \mathbf{F}\boldsymbol{C} \to \mathscr{C}$. We write $D(f) = D_f$. If $a : C \to D$, $f : A \to C$ and $g : D \to B$ are morphisms in $\boldsymbol{C}$, then the morphism $D_f \to D_{af}$ induced by the morphism $(1_A, a) : f \to af$ in $\mathbf{F}\boldsymbol{C}$ will be denoted by a_*, while the morphism $D_g \to D_{ga}$ induced by $(a, 1_B) : g \to ga$ will be denoted by a^*.

A morphism of natural systems is just a natural transformation. For a functor $q : \boldsymbol{C}' \to \boldsymbol{C}$, any natural system D on $\boldsymbol{C}$ gives a natural system $D \circ (\mathbf{F}q)$ on $\boldsymbol{C}'$ which we will denote $q^*(D)$.

For us a crucial observation is that any 2-category gives rise to a natural system. Indeed let $\mathscr{B}$ be a 2-category. There is a natural system $\mathrm{End}_{\mathscr{B}}$ of monoids on $\mathscr{B}_0$ (i. e. a functor $\mathbf{F}\mathscr{B}_0 \to$ **Monoids**) which assigns to an 1-arrow $f : A \to B$ the monoid of all 2-arrows $f \Rightarrow f$ in $\mathscr{B}$. Indeed for $g : B \to B'$, $h : A' \to A$ morphisms in $\mathscr{B}_0$ we already defined the induced homomorphisms:

$$(\varepsilon \mapsto g_*\varepsilon = g\varepsilon) : \mathrm{Hom}_{\mathscr{B}}(f, f) \to \mathrm{Hom}_{\mathscr{B}}(gf, gf),$$
$$(\varepsilon \mapsto h^*\varepsilon = \varepsilon h) : \mathrm{Hom}_{\mathscr{B}}(f, f) \to \mathrm{Hom}_{\mathscr{B}}(fh, fh).$$

For a track category $\mathscr{T}$, clearly $\mathrm{End}_{\mathscr{T}} = \mathrm{Aut}_{\mathscr{T}}$ takes values in the category of groups.

3. $\mathscr{T}$-natural systems

To state our main result we need to introduce some more notions.

3.1. Definition. *Consider a track category $\mathscr{T}$. A $\mathscr{T}$-*natural system *with values in a category $\mathscr{C}$ is a natural system $D : \mathbf{F}\mathscr{T}_0 \to \mathscr{C}$ on $\mathscr{T}_0$ together with a family of morphisms*

$$\nabla_\xi : D_f \to D_g$$

in the category $\mathscr{C}$, one for each track $\xi : f \Rightarrow g$ in $\mathscr{T}$, such that the following conditions are satisfied:

i) $\nabla_{0_f} = \mathrm{id}_{D_f}$ for all 1-arrows f in $\mathscr{T}$.

ii) For $\xi : f \Rightarrow g$, $\eta : g \Rightarrow h$ one has $\nabla_{\eta+\xi} = \nabla_\eta \circ \nabla_\xi$.

iii) For a diagram

$$\begin{tikzcd} \bullet & \bullet \arrow[l, "f"'] & \bullet \arrow[l, bend right, "g"'{name=U}] \arrow[l, bend left, "g_1"{name=L}] \arrow[from=U, to=L, phantom, "\Downarrow\xi"] & \bullet \arrow[l, "h"'] \end{tikzcd}$$

the following diagram

$$\begin{tikzcd}
D_{fg} \arrow[d, "\nabla_{f\xi}"'] & D_g \arrow[l, "f_*"'] \arrow[r, "h^*"] \arrow[d, "\nabla_\xi"'] & D_{gh} \arrow[d, "\nabla_{\xi h}"'] \\
D_{fg_1} & D_{g_1} \arrow[l, "f_*"'] \arrow[r, "h^*"] & D_{g_1 h}
\end{tikzcd}$$

commutes.

iv) *For a diagram*

$$\begin{tikzcd}
\bullet & \bullet \arrow[l, bend right, "f"'] \arrow[l, bend left, "f_1"] \arrow[l, phantom, "\Downarrow\xi"] & \bullet \arrow[l, "g"'] & \bullet \arrow[l, bend right, "h"'] \arrow[l, bend left, "h_1"] \arrow[l, phantom, "\Downarrow\eta"]
\end{tikzcd}$$

the diagram

$$\begin{tikzcd}
D_{fg} \arrow[dd, "\nabla_{\xi g}"'] & & D_{gh} \arrow[dd, "\nabla_{g\eta}"] \\
& D_g \arrow[ul, "f_*"] \arrow[ur, "h^*"'] \arrow[dl, "{f_1}_*"'] \arrow[dr, "{h_1}^*"] & \\
D_{f_1 g} & & D_{gh_1}
\end{tikzcd}$$

commutes.

A morphism $\varPhi : (D, \nabla) \to (D', \nabla')$ of $\mathscr{T}$-natural systems is a natural transformation $\varPhi$ between the functors $D, D' : \mathbf{F}\mathscr{T}_0 \to \mathscr{C}$, such that the diagram

$$\begin{tikzcd}
D \circ \mathbf{F}s \arrow[r, "\varPhi\mathbf{F}s"] \arrow[d, "\nabla"'] & D' \circ \mathbf{F}s \arrow[d, "\nabla'"] \\
D \circ \mathbf{F}t \arrow[r, "\varPhi\mathbf{F}t"] & D' \circ \mathbf{F}t
\end{tikzcd}$$

commutes. We denote by $\mathscr{T}$-Nat *the category of $\mathscr{T}$-natural systems.*

Let $G : \mathscr{T}' \to \mathscr{T}$ be a track functor. For any $\mathscr{T}$-natural system (D, ∇) one defines a $\mathscr{T}'$-natural system $G^*(D, \nabla) = (D \circ \mathbf{F}G, \nabla G)$, where for $\xi' : f' \Rightarrow g'$ in $\mathscr{T}'$, $(\nabla G)_{\xi'} : D_{Gf'} \to D_{Gg'}$ is defined to be $\nabla_{G\xi'}$. In this way one obtains a functor

$$G^* : \mathscr{T}\text{-Nat} \to \mathscr{T}'\text{-Nat}.$$

3.2. Example. *For a track category $\mathscr{T}$, the group-valued natural system* $\mathrm{Aut}_{\mathscr{T}}$ *is equipped with a canonical structure of a $\mathscr{T}$-natural system given by*

$$\nabla_\xi(a) = \xi + a - \xi.$$

Let D be a natural system on $\mathscr{T}_{\simeq}$. Then q^*D is a natural system on $\mathscr{T}_0$ given by $(q^*D)_f = D_{q(f)}$. Here $q : \mathscr{T}_0 \to \mathscr{T}_{\simeq}$ is the canonical projection. Define the structure of a $\mathscr{T}$-natural system on q^*D by $\nabla = \mathrm{id} : D \circ \mathbf{F}q \circ \mathbf{F}s = D \circ \mathbf{F}q \circ \mathbf{F}t$. In this way one obtains the functor $q^* : \mathrm{Nat}(\mathscr{T}_{\simeq}) \to \mathscr{T}$-Nat. Our Theorem 4.1 claims that the functor q^* is a full embedding. Actually we also identify the essential image of the functor q^*. We need the following definition.

3.3. Definition. *A $\mathscr{T}$-natural system (D,∇) is called* inert *if $\nabla_\varepsilon = \mathrm{id}_f$ for all $\varepsilon : f \Rightarrow f$.*

Inert $\mathscr{T}$-natural systems form a full subcategory of the category of $\mathscr{T}$-natural systems, which is denoted by $\mathscr{T}$-Inert. It is clear that the image of the functor q^* lies in $\mathscr{T}$-Inert. It is also clear that $\mathrm{Aut}_{\mathscr{T}}$ equipped with the canonical $\mathscr{T}$-natural system structure defined in Example 3.2 is inert if and only if $\mathscr{T}$ is an abelian track category.

Let us observe that for any track functor $G : \mathscr{T}' \to \mathscr{T}$ restriction of the functor $G^* : \mathscr{T}\text{-Nat} \to \mathscr{T}'\text{-Nat}$ yields the functor $G^* : \mathscr{T}\text{-Inert} \to \mathscr{T}'\text{-Inert}$.

4. The main result

4.1. Theorem. *Let $\mathscr{T}$ be a track category. Then $q^* : \mathrm{Nat}(\mathscr{T}_\simeq) \to \mathscr{T}$-Inert is an equivalence of categories. Furthermore, for any track functor $G : \mathscr{T}' \to \mathscr{T}$ the diagram*

$$\begin{tikzcd}
\mathrm{Nat}(\mathscr{T}_\simeq) \arrow[r, "q^*"] \arrow[d, "G^*_\simeq"] & \mathscr{T}\text{-Inert} \arrow[d, "G^*"] \\
\mathrm{Nat}(\mathscr{T}'_\simeq) \arrow[r, "q'^*"] & \mathscr{T}'\text{-Inert}
\end{tikzcd}$$

commutes.

Proof. Let E and E' be natural systems on $\mathscr{T}_\simeq$ and let $\Phi : q^*E \to q^*E'$ be a morphism of $\mathscr{T}$-natural systems. We claim that if f and g are homotopic maps in $\mathscr{T}_0$ (and therefore $qf = qg$), then the homomorphisms $\Phi_f : E_{qf} \to E'_{qf}$ and $\Phi_g : E_{qg} \to E'_{qg}$ are the same. Indeed, we can choose a track $\xi : f \Rightarrow g$. Then we have the following commutative diagram:

$$\begin{tikzcd}
(q^*E)_f \arrow[r, "\nabla_\xi"] \arrow[d, "\Phi_f"'] & (q^*E)_g \arrow[d, "\Phi_g"] \\
(q^*E')_f \arrow[r, "\nabla'_\xi"'] & (q^*E')_g
\end{tikzcd}$$

By definition of the $\mathscr{T}$-natural system structure on q^*E and q^*E' the morphisms ∇_ξ and ∇'_ξ are the identity morphisms, hence the claim. This shows that the functor q^* is full and faithful.

It remains to show that for any inert $\mathscr{T}$-natural system (D,∇) there exists a natural system E on $\mathscr{T}_\simeq$ and an isomorphism $\Delta : D \to q^*E$ of $\mathscr{T}$-natural systems. First of all one observes that if $\xi, \eta : f \Rightarrow g$ are tracks, then $\nabla_\xi = \nabla_\eta : D_f \to D_g$. Indeed, thanks to the property ii) of Definition 3.1 we have

$$\nabla_\xi = \nabla_{\xi-\eta+\eta} = \nabla_{\xi-\eta}\nabla_\eta = \nabla_\eta,$$

because $\xi - \eta : g \Rightarrow g$ and D is inert. Therefore for $qf = qg$ there is a well defined homomorphism $\nabla_{f,g} : D_f \to D_g$ induced by any track $f \Rightarrow g$. Then the relation ii) of Definition 3.1 shows that $\nabla_{g,h}\nabla_{f,g} = \nabla_{f,h}$ for any composable 1-arrows f, g, h. By harmless abuse of notation we will just write ∇ instead of $\nabla_{f,g}$ in what follows.

Since the functor $q : \mathscr{T}_0 \to \mathscr{T}_{\simeq}$ is identity on objects and full, we can choose for any arrow a in $\mathscr{T}_{\simeq}$ a map $u(a)$ in $\mathscr{T}_0$ such that $qu(a) = a$. Moreover for any map f in $\mathscr{T}_0$ we can choose a track $\delta(f) : f \Rightarrow u(qf)$. Now we put

$$E_a := D_{u(a)} \text{ and } \Delta_f := \nabla = \nabla_{f,u(qf)} = \nabla_{\delta(f)} : D_f \to D_{u(qf)} = E_{qf}.$$

For a diagram $\xleftarrow{c}\xleftarrow{a}\xleftarrow{b}$ in the category $\mathscr{T}_{\simeq}$ we define the homomorphism $c_* : E_a \to E_{ca}$ to be the following composite:

$$E_a = D_{u(a)} \xrightarrow{u(c)_*} D_{u(c)u(a)} \xrightarrow{\nabla} D_{u(ca)} = E_{ca}.$$

Similarly we define the homomorphisms $b^* : E_a \to E_{ab}$ to be the following composites:

$$E_a = D_{u(a)} \xrightarrow{u(b)^*} D_{u(a)u(b)} \xrightarrow{\nabla} D_{u(ab)} = E_{ab}.$$

It follows from the property iii) of Definition 3.1 that for any diagram $\xleftarrow{c_1}\xleftarrow{c}\xleftarrow{a}$ in the category $\mathscr{T}_{\simeq}$ we have the following commutative diagram:

$$\begin{tikzcd}
D_{u(a)} \arrow[d, "u(c)_*"'] \arrow[dr, "c_*"] & & \\
D_{u(c)u(a)} \arrow[r, "\nabla"] \arrow[d, "u(c_1)_*"'] & D_{u(ca)} \arrow[d, "u(c_1)_*"'] \arrow[dr, "c_{1*}"] & \\
D_{u(c_1)u(c)u(a)} \arrow[r, "\nabla"'] & D_{u(c_1)u(ca)} \arrow[r, "\nabla"'] & D_{u(c_1ca)}
\end{tikzcd}$$

Thus $c_1(c_*) = \nabla(u(c_1)(u(c)_*))$. On the other hand by definition we have the commutative diagram:

$$\begin{tikzcd}
D_{u(a)} \arrow[d, "u(c_1c)_*"'] \arrow[dr, "(c_1c)_*"] & \\
D_{u(c_1c)u(a)} \arrow[r, "\nabla"] & D_{u(c_1ca)}
\end{tikzcd}$$

It follows from the property iv) of Definition 3.1 that one has also the following commutative diagram

$$\begin{tikzcd}
D_{u(a)} \arrow[r, "(u(c_1)u(c))_*"] \arrow[d, "u(c_1c)_*"'] & D_{u(c_1)u(c)u(a)} \arrow[dl, "\nabla"] \arrow[d, "\nabla"'] \\
D_{u(c_1c)u(a)} \arrow[r, "\nabla"'] & D_{u(c_1ca)}
\end{tikzcd}$$

Therefore

$$(c_1c)_* = \nabla(u(c_1c)_*) = \nabla(\nabla(u(c_1)(u(c)_*))) = c_1(c_*).$$

Similarly $(b_1 b)^* = (b_1{}^*)b$ and E is a well-defined natural system on $\mathscr{T}_{\simeq}$. It remains to show that $\Delta : D \to q^*E$ is a natural transformation of functors defined on $\mathbf{F}\mathscr{T}_0$. To this end, one observes that for any composable morphisms g, f in the category $\mathscr{T}_0$ we have the following commutative diagram

$$\begin{tikzcd}
D_f \arrow[r, "\nabla"] \arrow[d, "g_*"] & D_{u(qf)} \arrow[d, "g_*"] \arrow[dr, "u(qg)_*"] & & \\
D_{gf} \arrow[r, "\nabla"] \arrow[rrr, bend right, "\nabla"'] & D_{gu(qf)} \arrow[r, "\nabla"] & D_{u(qg)u(qf)} \arrow[r, "\nabla"] & D_{uq(gf)}
\end{tikzcd}$$

This means that the following diagram also commutes:

$$\begin{tikzcd}
D_f \arrow[r, "\Delta_f"] \arrow[d, "g_*"] & E_{qf} \arrow[d, "(qg)_*"] \\
D_{gf} \arrow[r, "\Delta_{gf}"'] & E_{q(gf)}
\end{tikzcd}$$

Similarly the diagram

$$\begin{tikzcd}
D_g \arrow[r, "\Delta_g"] \arrow[d, "f^*"] & E_{qg} \arrow[d, "(qf)^*"] \\
D_{gf} \arrow[r, "\Delta_{gf}"'] & E_{q(gf)}
\end{tikzcd}$$

also commutes and therefore Δ is indeed a natural transformation. ■

Now let $\mathscr{T}$ be an abelian track category, so that $\mathrm{Aut}_{\mathscr{T}}$ is a natural system on $\mathscr{T}_0$ with values in the category of abelian groups. According to Example 3.2 it is equipped with the canonical structure of a $\mathscr{T}$-natural system, which is moreover inert, because $\mathscr{T}$ is abelian. Thus one can use Theorem 4.1 to conclude that there is a natural system D defined on $\mathscr{T}_{\simeq}$ and an isomorphism of $\mathscr{T}$-natural systems $\tau : \mathrm{Aut}_{\mathscr{T}} \to q^*D$ defined on $\mathscr{T}_0$. This was the main result of [Baues & Jibladze 2002].

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