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PROTOMODULARITY, DESCENT, AND SEMIDIRECT PRODUCTS

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Transmitted by Peter Johnstone

ABSTRACT. Using descent theory we give various forms of short five-lemma in protomodular categories, known in the case of exact protomodular categories. We also describe the situation where the notion of a semidirect product can be defined categorically.

Introduction

Protomodular categories were introduced in [B1]; they are closely related to the modular categories in the sense of A. Carboni [C], the Mal'tsev categories in the sense of A. Carboni, J. Lambek and M.C. Pedicchio [CLP] and the naturally Mal'tsev categories in the sense of P.T. Johnstone [Jo] — the precise relationship (modular $\Rightarrow$ naturally Mal'tsev $\Rightarrow$ (provided it has a zero object) protomodular, and protomodular $\Rightarrow$ Mal'tsev) is described in [B1] and [B2].

The protomodular categories are convenient for Nonabelian Homological Algebra : an exact category with zero object is protomodular if and only if it satisfies the "short five-lemma".

This note is based on two new observations on protomodular categories :

1°. Using descent theory one can generalize various forms of the short five-lemma to non-exact categories, which again give conditions equivalent to protomodularity.

2°. The pullback functor $p^* : Pt(B) \to Pt(E)$ associated with a morphism $p : E \to B$ in many important cases is monadic — not just conservative as required in the definition of a protomodular category. This has an important application : we obtain a categorical notion of a semidirect product which generalizes the algebraic semidirect products (although we consider below only the example of groups).

There are four sections :

1. *General protomodular categories* gives various conditions equivalent to protomodularity in a very general situation where only the existence of pullbacks is required.

2. *Pointed protomodular categories* gives further equivalent conditions under the existence of a zero object.

3. *Monadicity of $p^* : Pt(B) \to Pt(E)$ and semidirect products* describes the situation where semidirect products can be defined categorically.

4. *Semidirect product of groups* : we briefly show how can we obtain the classical semidirect product of groups as a special case.

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The terminology and simple facts from descent theory which we freely use in the first two sections can be found in [JaT1] and [JaT2].

1. General protomodular categories

Let $\mathbb{C}$ be a category with pullbacks and $Pt : \mathbb{C}^{op} \to \mathrm{Cat}$ the pseudofunctor of pointed objects. Recall that for an object C in $\mathbb{C}$, $Pt(C)$ is the category of triples (A, α, β) consisting of an object A in $\mathbb{C}$ and morphisms $\alpha : A \to C$, $\beta : C \to A$ with $\alpha\beta = 1_C$.

We will also use the pseudofunctors $C \mapsto (\mathbb{C} \downarrow C)$ and $C \mapsto (\mathbb{C} \Downarrow C)$ (again from $\mathbb{C}^{op}$ to Cat), where $(\mathbb{C} \downarrow C)$ is the usual comma category (= slice category) over C, and $(\mathbb{C} \Downarrow C)$ its full subcategory with objects all pairs (A, α) in which $\alpha : A \to C$ is an effective descent morphism, i.e. the pullback functor $(\mathbb{C} \downarrow C) \to (\mathbb{C} \downarrow A)$ is monadic.

For a given morphism $p : E \to B$ in $\mathbb{C}$ it is convenient to write p^* for each of the three pullback functors

$$(\mathbb{C} \downarrow B) \longrightarrow (\mathbb{C} \downarrow E),$$
$$(\mathbb{C} \Downarrow B) \longrightarrow (\mathbb{C} \Downarrow E),$$
$$Pt(B) \longrightarrow Pt(E).$$

1.1. Definition. [B1] *A category $\mathbb{C}$ is said to be protomodular if the functor $p^* : Pt(B) \to Pt(E)$ reflects isomorphisms for every morphism $p : E \to B$ in $\mathbb{C}$.*

Recall ([B1], [B2]) that among the examples of protomodular categories are the varieties of weakly associative Mal'tsev algebras (i.e. the varieties which have a ternary operator p with $p(x, x, z) = z$, $p(x, z, z) = x$, $p(p(x, y, z), z, y) = x$), in particular groups, rings, etc., the category of Heyting algebras, the dual of any elementary topos, and in particular the dual of the category of sets.

Consider a commutative diagram in $\mathbb{C}$ of the form

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "\alpha"'] & A' \arrow[r, "f'"] \arrow[d, "\alpha'"'] & A'' \arrow[d, "\alpha''"'] \\ B \arrow[r, "g"] & B' \arrow[r, "g'"] & B'' \end{tikzcd} \qquad (1)$$

in which

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "\alpha"'] & A' \arrow[d, "\alpha'"] \\ B \arrow[r, "g"'] & B' \end{tikzcd} \quad \text{and} \quad \begin{tikzcd} A \arrow[r, "f'f"] \arrow[d, "\alpha"'] & A'' \arrow[d, "\alpha''"] \\ B \arrow[r, "g'g"'] & B'' \end{tikzcd} \qquad (2)$$

are pullbacks. Then we will say that (1) is a *pullback cancellation diagram.* If the right hand square in (1) is also a pullback, then we say that the diagram (1) *admits cancellation.*

1.2. THEOREM. *Let $\mathbb{C}$ be a category with pullbacks. The following conditions are equivalent :*

(a) $\mathbb{C}$ is protomodular;

(b) the functor $p^* : (\mathbb{C} \Downarrow B) \to (\mathbb{C} \Downarrow E)$ reflects isomorphisms for every morphism $p : E \to B$ in $\mathbb{C}$;

(c) every pullback cancellation diagram (1) in $\mathbb{C}$, in which α' is a regular epimorphism in $(\mathbb{C} \downarrow B'')$ and α'' an effective descent morphism, admits cancellation;

(d) the same as (c), but α' is required to be regular epimorphism in $\mathbb{C}$;

(e) the same as (c), but the diagram (1) is required to be split, i.e. there exist morphisms $\beta : B \to A$, $\beta' : B' \to A$, $\beta'' : B'' \to A''$ with $\alpha\beta = 1_B$, $\alpha'\beta' = 1_{B'}$, $\alpha''\beta'' = 1_{B''}$;

(f) the same as (c), but g' is required to be an isomorphism;

(g) the same as (d), but g' is required to be an isomorphism;

(h) the same as (e), but g' is required to be an isomorphism.

Proof. Since

(i) every morphism in $(\mathbb{C} \downarrow B'')$ which is a regular epimorphism in $\mathbb{C}$, is also a regular epimorphism in $(\mathbb{C} \downarrow B'')$;

(ii) every split epimorphism is an effective descent morphism,

the implications

$$\begin{array}{ccccccc} (c) & \Longrightarrow & (d) & \Longrightarrow & (e) & & (b) \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\ (f) & \Longrightarrow & (g) & \Longrightarrow & (h) & \Longleftrightarrow & (a) \end{array}$$

are trivial. Moreover, $(a) \Leftrightarrow (e)$ by [B1, Proposition 7], and $(b) \Leftrightarrow (f)$ can be shown with the same arguments using the fact that every effective descent morphism $A' \to B'$ is a regular epimorphism in $(\mathbb{C} \downarrow B')$.

Therefore we need to prove only $(e) \Rightarrow (c)$.

For a given diagram (1) as in (c) consider the diagram

$$\begin{array}{ccccc} A\times_B A & \xrightarrow{f\times f} & A'\times_{B'}A' & \xrightarrow{f'\times f'} & A''\times_{B''}A'' \\ \Big\downarrow{\scriptstyle pr_1} & & \Big\downarrow{\scriptstyle pr_1} & & \Big\downarrow{\scriptstyle pr_1} \\ A & \xrightarrow[f]{} & A' & \xrightarrow[f']{} & A'' \end{array}$$

this diagram is split, and so its right hand square is a pullback by (e). Therefore

$$\begin{tikzcd} A'\times_{B'}A' \arrow[r, shift left] \arrow[r, shift right] \arrow[d, "f'\times f'"'] & A' \arrow[d, "f'"] \\ A''\times_{B''}A'' \arrow[r, shift left] \arrow[r, shift right] & A'' \end{tikzcd}$$

is an internal discrete (op)fibration, and since α'' is an effective descent morphism, it corresponds to a unique (up to an isomorphism) object in $(\mathbb{C} \downarrow B'')$. Furthermore, since α' is a regular epimorphism in $(\mathbb{C} \downarrow B'')$, that unique object is (B', g'), and so the right hand square in (1) is a pullback as desired. ∎

2. Pointed protomodular categories

A category $\mathbb{C}$ with pullbacks is said to be *pointed* if it has a zero objet 0. In such a category every morphism $\alpha : A \to B$ has a *kernel* $\kappa : K \to A$ which can be defined as a morphism for which the diagram

$$\begin{tikzcd} K \arrow[r, "\kappa"] \arrow[d] & A \arrow[d, "\alpha"] \\ 0 \arrow[r] & B \end{tikzcd}$$

is a pullback.

Let us recall the following well known

2.1. Lemma. *Let*

$$\begin{tikzcd} K \arrow[r, "\kappa"] \arrow[d, "k"'] & A \arrow[r, "\alpha"] \arrow[d, "f"'] & B \arrow[d, "g"'] \\ K' \arrow[r, "\kappa'"'] & A' \arrow[r, "\alpha'"'] & B' \end{tikzcd} \qquad (3)$$

be a commutative diagram in a pointed category $\mathbb{C}$,*in which the right hand square is a pullback,* κ *is a kernel of* α*, and* κ' *is a kernel of* α'*. Then* k *is an isomorphism.*

It is convenient to have a special name for a commutative diagram (3) in which κ is a kernel of α, κ' is a kernel of α', and k is an isomorphism; we will call it a *kernel isomorphism diagram.*

2.2. Remark. (a) Let

$$\begin{tikzcd}
 & K \arrow[r] \arrow[d, "\kappa"'] \arrow[dr, phantom, "1"] & 0 \arrow[d] \\
K \arrow[r, "\kappa"] \arrow[d, "k"'] \arrow[dr, phantom, "2"] & A \arrow[r, "\alpha"] \arrow[d, "f"'] \arrow[dr, phantom, "3"] & B \arrow[d, "g"'] \\
K' \arrow[r, "\kappa'"'] & A' \arrow[r, "\alpha'"'] & B'
\end{tikzcd}$$

be a commutative diagram in which κ is a kernel of α and κ' is a kernel of α'. Then the squares 2 and 3 form a kernel isomorphism diagram if and only if the squares 1 and 3 form a pullback cancellation diagram — which simply means that $f\kappa$ is a kernel of α'.

(b) Let $i_B : 0 \to B$ be the unique morphism from 0 to $B \in \mathbb{C}$. If $\mathbb{C}$ is pointed, then it is protomodular if and only if the functor $i_B^* : Pt(B) \to Pt(0) \simeq \mathbb{C}$ reflects isomorphisms for every object B in $\mathbb{C}$. This follows from the fact for every $p : E \to B$ in $\mathbb{C}$, we have $pi_E = i_B$ and therefore $i_B^* \simeq i_E^* p^*$.

(c) Let us recall [B1] that if $\mathbb{C}$ is pointed and protomodular, then a morphism $\alpha : A \to B$ is a monomorphism if and only if $i_A : 0 \to A$ is a kernel of α.

(d) Let us also recall that in a pointed exact category the short five lemma says that in every kernel isomorphism diagram (3), in which α and α' are regular epimorphisms and g is an isomorphism, f also is an isomorphism. This lemma holds for example for groups, but not for monoids.

2.3. Theorem. *Let $\mathbb{C}$ be a pointed category (with pullbacks, as above). The following conditions are equivalent :*

(a) $\mathbb{C}$ is protomodular;

(b) the functor $i_B^* : (\mathbb{C} \Downarrow B) \to (\mathbb{C} \Downarrow O) \simeq \mathbb{C}$ reflects isomorphisms for every objet B in $\mathbb{C}$;

(c) in every kernel isomorphism diagram (3) *in which α is a regular epimorphism in $(\mathbb{C} \downarrow B')$ and α' is an effective descent morphism, the right hand square is a pullback;*

(d) the same as (c), but α is required to be a regular epimorphism in $\mathbb{C}$;

(e) the same as (c), but the right hand square in (3) *is required to be split, i.e. there exist $\beta : B \to A$, $\beta' : B' \to A'$ with $\alpha\beta = 1_B$, $\alpha'\beta' = 1_{B'}$.*

(f) ("generalized short five lemma") in every kernel isomorphism diagram (3)*, in which α is a regular epimorphism in $(\mathbb{C} \downarrow B')$, α' is an effective descent morphism, and g is an isomorphism, f also is an isomorphism;*

(g) the same as (f), but α is required to be a regular epimorphism in $\mathbb{C}$;

(h) ("split short five lemma") the same as (f), but the right hand square in (3) *is required to be split as in (e).*

Proof. Just as in the proof of theorem 1.2, it is easy to see that the condition (h) follows from any other condition above. On the other hand the remark 2.2(a) tells us that any of these conditions follow from the corresponding conditions in Theorem 1.2. Therefore it suffices to show that (h) implies protomodularity, but this immediately follows from the remark 2.2(b). ∎

2.4. Corollary. *Let $\mathbb{C}$ be a pointed category. The following conditions are equivalent :*

(a) $\mathbb{C}$ is protomodular;

(b) in every kernel isomorphism diagram (3) *in which α and α' are effective descent morphisms and g is an isomorphism, f also is an isomorphism;*

(c) in every kernel isomorphism diagram (3) *in which α and α' are split epimorphisms and g is an isomorphism, f also is an isomorphism.*

3. Monadicity of $p^* : Pt(B) \to Pt(E)$ and semidirect products

In this section $\mathbb{C}$ denotes an arbitrary category; we do *not* even assume that $\mathbb{C}$ has pullbacks.

3.1. Definition. *(a) A diagram*

$$\begin{tikzcd} D \arrow[r, "q"] \arrow[d, shift left, "\delta"] & A \arrow[d, shift left, "\alpha"] \\ E \arrow[r, "p"'] \arrow[u, shift left, "\epsilon"] & B \arrow[u, shift left, "\beta"] \end{tikzcd} \qquad (4)$$

is said to be a split commutative square *if $\alpha\beta = 1_B$, $\delta\epsilon = 1_E$, $\alpha q = p\delta$ and $q\epsilon = \beta p$;*

(b) a split commutative square (4) *is said to be a* split pullback*, if for every split commutative square of the form*

$$\begin{tikzcd} D' \arrow[r, "q'"] \arrow[d, shift left, "\delta'"] & A \arrow[d, shift left, "\alpha"] \\ E \arrow[r, "p"'] \arrow[u, shift left, "\epsilon'"] & B \arrow[u, shift left, "\beta"] \end{tikzcd}$$

there exists a unique morphism $d : D' \to D$ with $\delta d = \delta'$, $d\epsilon' = \epsilon$ and $qd = q'$;

(c) we say that $\mathbb{C}$ has split pullbacks *if for every morphism $p : E \to B$ in $\mathbb{C}$ and every $(A, \alpha, \beta) \in Pt(B)$ there exists a split pullback of the form* (4)*;*

(d) dually, a split commutative square (4) *is said to be a* split pushout *if for every*

split commutative square of the form

$$\begin{tikzcd} D \arrow[r, "q'"] \arrow[d, shift left, "\delta"] & A' \arrow[d, shift left, "\alpha'"] \\ E \arrow[r, "p"'] \arrow[u, shift left, "\epsilon"] & B \arrow[u, shift left, "\beta'"] \end{tikzcd}$$

there exists a unique morphism $a : A \to A'$ *with* $\alpha' a = \alpha$, $a\beta = \beta'$ *and* $aq = q'$*;*

(e) we say that $\mathbb{C}$ has split pushouts *if for every morphism* $p : E \to B$ *in* $\mathbb{C}$ *and every* $(D, \delta, \epsilon) \in Pt(E)$ *there exists a split pushout of the form* (4)*.*

Note that the pullbacks of split epimorphisms are split pullbacks, but the existence of split pullbacks does not imply the existence of pullbacks of split epimorphisms.

For example, if $\mathbb{C}$ is the category generated by the graph

$$\begin{tikzcd} A \arrow[r, shift right, "\alpha"'] & B \arrow[l, shift right, "\beta"'] \end{tikzcd}$$

and the identity $\alpha\beta = 1_B$, then $\mathbb{C}$ has split pullbacks one of which is not obtained as an ordinary pullback — this is

$$\begin{tikzcd} A \arrow[r, "\beta\alpha"] \arrow[d, shift left, "1_A"] & A \arrow[d, shift left, "\alpha"] \\ A \arrow[r, "\alpha"'] \arrow[u, shift left, "1_A"] & B \arrow[u, shift left, "\beta"] \end{tikzcd}$$

However the existence of split pullbacks allows us to define a functor

$$p^* : Pt(B) \to Pt(E), \tag{5}$$

for every $p : E \to B$ in $\mathbb{C}$, and it can be considered as a generalization of the ordinary pullback functor p^* considered in the previous sections.

Note also, that the functors (5) have left adjoints (for all p) if and only if $\mathbb{C}$ has split pushouts.

3.2. Definition. *A category* $\mathbb{C}$ *with split pullbacks is said to be a* category with semidirect products *if, for every morphism* $p : E \to B$ *in* $\mathbb{C}$*, the functor* (5) *has a left adjoint and is monadic.*

Let T^p be the monad on $Pt(E)$ corresponding to the (monadic) functor (5). Given a T^p-algebra (X, ξ), we define the *semidirect product*

$$(X, \xi) \rtimes (B, p)$$

to be the object in $Pt(B)$ corresponding to (X, ξ) under the category equivalence

$$Pt(B) \sim Pt(E)^{T^p}.$$

In particular, if $\mathbb{C}$ is a pointed category and $p = i_B : 0 \to B$, we could write $T^p = T^B$ and

$$(X, \xi) \rtimes (B, i_B) = (X, \xi) \rtimes B. \tag{6}$$

Note that in this case $Pt(E) = Pt(0)$ can be identified with $\mathbb{C}$, and so X is just an object in $\mathbb{C}$.

3.3. Example. If $\mathbb{C}$ is *additive*, then T^B is just the identity monad on $\mathbb{C}$ and we can write $(X, \xi) \rtimes B = X \rtimes B$. In fact in this case $X \rtimes B = X \oplus B$, i.e. $X \rtimes B$ is the direct sum of X and B.

As we will see in the next section, the ordinary semidirect product of groups is a special case of (6), and so the category of groups is a category with semidirect products. The same is true for all algebraic protomodular categories, as follows from

3.4. Theorem. *(a) If $\mathbb{C}$ is an exact protomodular category, then it has coequalizers of reflexive pairs and the pullback functor $p^* : Pt(B) \to Pt(E)$ preserves such coequalizers for every morphism $p : E \to B$ in $\mathbb{C}$;*

(b) an exact category is a category with semidirect products if and only if it is protomodular and has pushouts of split monomorphisms.

Proof. (a) : Since $\mathbb{C}$ is exact, it has the coequalizers of equivalence relations preserved by the pullback functors. Since the pullbacks and coequalizers in $Pt(B)$ and $Pt(E)$ (which exist in $\mathbb{C}$) are "calculated as in $\mathbb{C}$", we conclude that $Pt(B)$ and $Pt(E)$ have coequalizers of equivalence relations and the functor p^* preserves them. On the other hand every reflexive pair can be replaced by a reflexive relation (as $\mathbb{C}$ is exact) with the same coequalizers, and this passage from the reflexive pairs to the reflexive relations is also preserved by the pullback functors. After that it suffices to recall [B2] that every protomodular category is a Mal'tsev category, and therefore every reflexive relation in $\mathbb{C}$ is an equivalence relation.

(b) : If we know that p^* has a left adjoint, then its monadicity follows from (a) and the "reflexive form" of Beck's Monadicity Theorem. The existence of a left adjoint for each p^* is equivalent to the existence of split pushouts, and, since the ground category $\mathbb{C}$ has all pullbacks, it is easy to show that the split pushouts are just the pushouts of split monomorphisms — provided all split pushouts exist. ∎

4. Semidirect product of groups

Let G be a group. Recall that a *G-group* is a pair (X, m) consisting of a group X and a map $m : G \times X \to X$ written as $(g, x) \mapsto gx$, such that

$$1x = x, \quad g'(gx) = (g'g)x,$$

$$g(xx') = (gx)(gx')$$

for every $x,\ x' \in X$ and $g,\ g' \in G$. In the category of groups there is a category equivalence

$$Pt(G) \sim G\text{-groups}$$

— it carries an object (A, α, β) of $Pt(G)$ to the pair (X, m) in which

$$X = \operatorname{Ker}\alpha = \{a \in A \mid \alpha(a) = 1\}$$

and $m : G \times X \to X$ is defined by

$$(g, x) \mapsto \beta(g)x\beta(g)^{-1}$$

— and conversely, a G-group (X, m) corresponds to the classical semidirect product

$$\begin{tikzcd} (1,g) & (X,m) \rtimes G \arrow[d, shift left] & (x,g) \arrow[d] \\ g \arrow[u] & G \arrow[u, shift left] & g \end{tikzcd}$$

Moreover, it is easy to see that the functor $i_G^* : Pt(G) \to Pt(0)$ considered in section 2 above, corresponds to the forgetful functor from the category of G-groups to the category of groups. Therefore T^G-algebras (see section 3) are the same as the G-groups - since the category of groups is a pointed exact protomodular category with pushouts.
Explicitly

$$T^G(X) = \text{ the kernel of } \begin{array}{l} G * X \to G \\ g \mapsto g\,(g \in G), \\ x \mapsto 1\,(x \in X), \end{array}$$

where $G * X$ is the free product (=categorical coproduct) of the groups G and X - and a T^G-algebra (X, ξ) corresponds to the G-group (X, m) in which $gx = m(g, x)$ is defined as

$$gx = \xi(g, x, g^{-1}).$$

Accordingly, the classical semidirect product of groups is a special case of the categorical one introduced in the previous section.

The same is true for more general semidirect products considered for example in [O].

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