ON A CONVERGENCE OF AN APPROXIMATE SOLUTION FOR A SOLID-FLUID MODEL

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This paper deals with the study of the approximation scheme for a boundary-contacts problem with unilateral contact conditions for the body containing parts of elastic solid and viscous fluid (see [1], [2]).

Let vector $h = (h_1, h_2, h_3) \in \mathbb{R}^3$. We define the mesh R_h (see [3])by

$$R_h = \{M | M \in \mathbb{R}^3, \;\; M = (m_1h_1, m_2h_2, m_3h_3), \;\; m_i \in Z\}. \tag{1}$$

To any node M of the mesh R_h we correspond a "bar" with the center M:

$$\omega_h^0(M) = \prod_{i=1}^{3}](m_i - \frac{1}{2})h_i, (m_i + \frac{1}{2})h_i[$$

and a "christ" with the center M:

$$\omega_h^1(M) = \bigcup_{i=1}^{3} \omega_h^0(M \pm \frac{h}{2}e_i),$$

where $e_i, \;\; i = 1, 2, 3$ are unit basis vectors in $\mathbb{R}^3$

Let $A = \{M \in R_h | dist(M, \sum) < \frac{1}{2}\sqrt{h_1^2 + h_2^2 + h_3^2}\}$. If $M \in A$, then we introduce $(\omega_h^0(M))' = \omega_h^0(M) \cap \Omega_s$ and $(\omega_h^0(M))'' = \omega_h^0(M) \cup \Omega_f$.

Now we define $\Omega_h = \{M | \omega_h^1(M) \subset \Omega\}$ and v_h which is the space streched on $\theta_h^M, \; (\theta_h^M)', \; (\theta_h^M)'', \;\; M \in \Omega_h$, where the last three symbols are characteristic functions respectively for $\omega_h^0(M)$, when $dist(M, \sum) \geq \frac{1}{2}\sqrt{h_1^2 + h_2^2 + h_3^2}$ and $(\omega_h^0(M))'$ and $(\omega_h^0(M))''$.

We assume that

$$K_h = \left\{ v_h | v_h \in V_h : (v_h, (\theta_h^M)') - (v_h, (\theta_h^M)'') \geq 0 \;\; a.e. \;\; on \sum \right\} \tag{2}$$

The following scheme of approximation has been built: find $u^2, u^3, ...u^n$, such, that

$$(\rho\gamma^i, v - d^i) + a(u^{i+1}, v - d^i) + b(d^i, v - d^i) \geq (f^i, v - d^i), \; (i \geq 1) \tag{3}$$

$$\begin{gathered} \forall v \in K_h, \;\; d^i \in K_h, \\ u^0 = u^1 = 0, \end{gathered} \tag{4}$$

where we use the following notations (we do not write h index):

$$\delta^i = \frac{u^{i+1} - u^i}{k}, \tag{5}$$

$$d^i = \frac{u^{i+1} - u^{i-1}}{2k} = \frac{\delta^i + \delta^{i-1}}{2}, \tag{6}$$

$$\gamma^i = \frac{\delta^i - \delta^{i-1}}{k} = \frac{u^{i+1} - 2u^i + u^{i-1}}{k^2} = 2\left(\frac{d^i - \delta^{i-1}}{k}\right), \tag{7}$$

$$g^i = \frac{f^{i+1} - f^i}{k}. \tag{8}$$

Taking into account these notations, we can rewrite the inequality (3) in the following way

$$(\rho d^i, v - d^i) - (\rho\delta^{i-1}, v - d^i) + k^2 a(d^i, v - d^i) + \frac{k}{2}a(u^i - k\delta^{i-1}, v - d^i) +$$

$$+\frac{k}{2}b(d^i, v - d^i) - \frac{k}{2}(f^i, v - d^i) \geq 0. \tag{9}$$

The problem (9), (4) can be reduced to the problem of minimization of the following functional:

$$J(d^i) = \frac{1}{2}\alpha|d^i|^2 + \frac{1}{2}k^2 a(d^i, d^i) + \frac{k}{4}b(d^i, d^i) + (\tilde{f}_i, d^i), \quad d_i \in K_h, \tag{10}$$

$$where\ (\tilde{f}_i, d^i) = -(\rho\delta^{i-1}, d^i) + \frac{k}{2}a(u^i - k\delta^{i-1}, d^i) - (f^i, d^i), \tag{11}$$

$$u^{i+1} = u^i + kd^i \tag{12}$$

Let $P_k : V_h \to K_h$ be a mapping defined as follows:

$$P_k(v_h) = \begin{cases} v_h \ \ if \ \ (v_h, (\theta_h^M)'') - (v_h, (\theta_h^M)') \geq 0, \ \ \forall M \in A, \\ v_h \ \ if \ \ (v_h, (\theta_h^M)'') - (v_h, (\theta_h^M)') < 0, \ \ \forall M \in A, \end{cases} \quad v_h \in V_h,$$

where $(v_h^1, \theta_h^M) = (v_h, \theta_h^M)$ if $M \in \Omega_h \backslash A$, and $(v_h^1, (\theta_h^M)') = (v_h^1, (\theta_h^M)'') = (v_h, \theta_h^M)'$ if $M \in A$.

Theorem. *Let*

$$d^0 \in K_h,\ d^{n+1} = P_k(d^n - \rho_n J'(d^n)), \tag{13}$$

where ρ_n are numbers.

We can choose the numbers ρ_0 and ρ_1 such, that if $0 < \rho_0 \leq \rho_n \leq \rho_1$, than the process of iteration (13) converges to the solution of the minimization problem (10).

R e f e r e n c e s

[1] G. Chichua. On a Boundary-contact Problem for a Solid-Fluid Model. Reports of Enlarged Session of the Seminar of I. Vekua Institute of Applied Mathematics, 1995, v. 10, $N^{\underline{o}}$1, p. 18-20.

[2] G. Chichua. Mixture of Elastic Solid and Viscous Fluid. Tbilisi International Centre of Mathematics and Informatics, 1997, v.1.

[3] Sanches-Palencia E. Non-Homogenous Media and Vibration Theory. Springer-Verlag, 1980.