EXTENDED ABSTRACTS OF MINISYMPOZIUM

ON ONE VARIANT OF THE IMPROVED, MORE ACCURATE THEORY OF LAMINATED ELASTIC ORTHOTROPIC SHELLS OF ROTATION IN UNSYMMETRIC DEFORMATIONS

E. Abramidze

N.I. Musckhelishvili Institute of Computational Mathematics of Georgian Academy of Sciences

For the multilayered shells, whose physical characters differ strongly from each other, the consideration of the deformation of transverse shear is essential. The application of the hypothesis of Kirchhof-Love and of rectilinearity of an element for the whole package of shell does not allow us to consider the inhomogeneity of deformation of transverse shear, caused by difference of elastic properties of layers.

In this note to overcome the indicated deficiency, the following approach is offered. It is based on the assumption of the presence of local corners in the rotation layers of the shell, caused by transverse shear, and on the satisfaction of the conditions of continuity of displacements and strains on the surface of contact of adjacent layers [1,2].

We shall write the basic equations of the theory of laminated orthotropic shells in geometrically nonlinear statement for quadratic approximation [3,4] taking into account the non-homogeneity of deformations of transverse shear in layers.

Denoting by $\gamma_\alpha^{(i)}$, $\gamma_\beta^{(i)}$ the transverse shears in the i-th layer, we obtain the expressions for the tangential displacements

$$\begin{aligned} u_\alpha^{(i)} &= \frac{1+k_1\gamma}{1+k_1\gamma_i}u^{(i)} - \frac{\gamma-\gamma_i}{A(1+k_1\gamma_i)}\frac{\partial\omega}{\partial\alpha} + (\gamma-\gamma_i)\gamma_\alpha^{(i)}, \\ u_\beta^{(i)} &= \frac{1+k_2\gamma}{1+k_2\gamma_i}v^{(i)} - \frac{\gamma-\gamma_i}{B(1+k_2\gamma_i)}\frac{\partial\omega}{\partial\beta} + (\gamma-\gamma_i)\gamma_\beta^{(i)}, \end{aligned} \tag{1}$$

where ω is a normal displacement, assumed unchanged in thickness; $u^{(i)}$, $v^{(i)}$ are tangential displacements of the surface $\gamma=\gamma_i$; A, B are the parameters of Lamé; k_1 k_2 are the main curvatures of the coordinate surface.

Assuming the continuity of the tangent strains $\tau_{\alpha\gamma}$, $\tau_{\beta\gamma}$, determined by the transverse shears $\varepsilon_{\alpha\gamma}=\gamma_\alpha$, $\varepsilon_{\beta\gamma}=\gamma_\beta$, we express $\gamma_\alpha^{(i)}$, $\gamma_\beta^{(i)}$, by $\gamma_\alpha^{(0)}$, $\gamma_\beta^{(0)}$, using Hooke's law in the form

$$\gamma_\alpha^{(i)} = \frac{G_{\alpha\gamma}^{(0)}}{G_{\alpha\gamma}^{(i)}}\gamma_\alpha^{(0)}, \quad \gamma_\beta^{(i)} = \frac{G_{\beta\gamma}^{(0)}}{G_{\beta\gamma}^{(i)}}\gamma_\beta^{(0)}, \tag{2}$$

where $G_{\alpha\gamma}^{(i)}$, $G_{\beta\gamma}^{(i)}$ are the modulus of the shear of the material in the i-th layer.

Satisfying the conditions of continuity of displacements, we express $u^{(i)}$, $v^{(i)}$ by the displacements of coordinate surface u, v and the total corners of rotation Ψ_α, Ψ_β in the layer with index $<< 0 >>$

$$\Psi_\alpha = -\frac{1}{A}\frac{\partial\omega}{\partial\alpha} + k_1 u + \gamma_\alpha^{(0)}, \quad \Psi_\beta = -\frac{1}{B}\frac{\partial\omega}{\partial\beta} + k_2 v + \gamma_\beta^{(0)}. \tag{3}$$

The indicated approach allows to represent the expressions of the tangential displacements (1) in the form

$$u_\alpha^{(i)} = u + a_1^{(i)}\gamma_\alpha^{(0)} + \gamma(\Psi_\alpha + a_2^{(i)}\gamma_\alpha^{(0)}), \quad u_\beta^{(i)} = v + b_1^{(i)}\gamma_\beta^{(0)} + \gamma(\Psi_\beta + b_2^{(i)}\gamma_\beta^{(0)}), \tag{4}$$

where $a_1^{(i)}$ $a_2^{(i)}$ $b_1^{(i)}$ $b_2^{(i)}$ are expressed by the values $G_{\beta\gamma}^{(i)}$ $G_{\beta\gamma}^{(i)}$.

Taking into account the expressions (4), we represent the components of the deformations of the shell in the form

$$\begin{gathered}\varepsilon_{\alpha\alpha}^{(\gamma)} = \varepsilon_{\alpha\alpha}^{(i)} + \gamma\varkappa_{\alpha\alpha}^{(i)},\ \varepsilon_{\beta\beta}^{(\gamma)} = \varepsilon_{\beta\beta}^{(i)} + \gamma\varkappa_{\beta\beta}^{(i)},\ \varepsilon_{\alpha\beta}^{(\gamma)} = \varepsilon_{\alpha\beta}^{(i)} + \gamma 2\varkappa_{\alpha\beta}^{(i)},\\ \varepsilon_{\alpha\gamma}^{(\gamma)} = \gamma_\alpha^{(i)},\ \varepsilon_{\beta\gamma}^{(\gamma)} = \gamma_\beta^{(i)},\ \varepsilon_{\gamma\gamma}^{(\gamma)} = 0.\end{gathered} \tag{5}$$

Here

$$\begin{gathered}\varepsilon_{\alpha\alpha}^{(i)} = \varepsilon_{\alpha\alpha} + \frac{1}{A}\frac{\partial a_1^{(i)}\gamma_\alpha^{(0)}}{\partial\alpha} + \frac{1}{AB}\frac{\partial A}{\partial\beta}b_1^{(i)}\gamma_\beta^{(0)},\ \varepsilon_{\beta\beta}^{(i)} = \varepsilon_{\beta\beta} + \frac{1}{B}\frac{\partial b_1^{(i)}\gamma_\beta^{(0)}}{\partial\beta} +\\ + \frac{1}{AB}\frac{\partial B}{\partial\alpha}a_1^{(i)}\gamma_\alpha^{(0)},\quad \varepsilon_{\alpha\beta}^{(i)} = \varepsilon_{\alpha\beta}^{*} + \frac{A}{B}\frac{\partial}{\partial\beta}\left(\frac{a_1^{(i)}\gamma_\alpha^{(0)}}{A}\right) + \frac{B}{A}\frac{\partial}{\partial\alpha}\left(\frac{b_1^{(i)}\gamma_\beta^{(0)}}{B}\right),\\[2ex] \varkappa_{\alpha\alpha}^{(i)} = \varkappa_\alpha + \frac{1}{A}\frac{\partial a_2^{(i)}\gamma_\alpha^{(0)}}{\partial\alpha} + \frac{1}{AB}\frac{\partial A}{\partial\beta}b_2^{(i)}\gamma_\beta^{(0)},\ \varkappa_{\beta\beta}^{(i)} = \varkappa_\beta + \frac{1}{B}\frac{\partial b_2^{(i)}\gamma_\beta^{(0)}}{\partial\beta} +\\ + \frac{1}{AB}\frac{\partial B}{\partial\alpha}a_2^{(i)}\gamma_\alpha^{(0)},\quad 2\varkappa_{\alpha\beta}^{(i)} = 2\varkappa_{\alpha\beta} + \frac{A}{B}\frac{\partial}{\partial\beta}\left(\frac{a_2^{(i)}\gamma_\alpha^{(0)}}{A}\right) + \frac{B}{A}\frac{\partial}{\partial\alpha}\left(\frac{b_2^{(i)}\gamma_\beta^{(0)}}{B}\right) -\\ - \frac{k_1}{A}\left[\frac{\partial}{\partial\alpha}\left(b_1^{(i)}\gamma_\beta^{(0)}\right) - \frac{1}{B}\frac{\partial A}{\partial\beta}\left(a_1^{(i)}\gamma_\alpha^{(0)}\right)\right] - \frac{k_2}{B}\left[\frac{\partial}{\partial\beta}\left(a_1^{(i)}\gamma_\alpha^{(0)}\right) - \frac{1}{A}\frac{\partial B}{\partial\alpha}b_1^{(i)}\gamma_\beta^{(0)}\right],\end{gathered} \tag{6}$$

where

$$\begin{gathered}\varepsilon_{\alpha\alpha} = \varepsilon_\alpha + \frac{1}{2}\theta_\alpha^2,\ \varepsilon_{\beta\beta} = \varepsilon_\beta + \frac{1}{2}\theta_\beta^2,\ \varepsilon_{\alpha\beta}^{*} = \varepsilon_{\alpha\beta} + \theta_\alpha\theta_\beta,\ \gamma_\alpha^{(0)} = \Psi_\alpha - \theta_\alpha,\\ \gamma_\beta^{(0)} = \Psi_\beta - \theta_\beta,\ \theta_\alpha = -\frac{1}{A}\frac{\partial\omega}{\partial\alpha} + k_1 u,\ \theta_\beta = -\frac{1}{B}\frac{\partial\omega}{\partial\beta} + k_2 v.\end{gathered} \tag{7}$$

The values ε_α, ε_β, $\varepsilon_{\alpha\beta}$, $\varkappa_\alpha$, $\varkappa_\beta$, $\varkappa_{\alpha\beta}$ characterize the deformation of coordinate surface of the shell. Their expressions are given in [2].

Having the expressions (5), (6), on the bases of Hooke's law we obtain the relations of elasticity

$$
\begin{aligned}
&N_\alpha = C_{11}\varepsilon_{\alpha\alpha} + C_{12}\varepsilon_{\beta\beta} + K_{11}\varkappa_\alpha + K_{12}\varkappa_\beta + A_{11}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha} + A_{12}\gamma_\alpha^{(0)} + \\
&+B_{11}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + B_{12}\gamma_\beta^{(0)}, \quad N_\beta = C_{12}\varepsilon_{\alpha\alpha} + C_{22}\varepsilon_{\beta\beta} + K_{12}\varkappa_\alpha + K_{22}\varkappa_\beta + \\
&+A_{21}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha} + A_{22}\gamma_\alpha^{(0)} + B_{22}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + B_{22}\gamma_\beta^{(0)}, \\
&N_{\alpha\beta} = C_{66}\varepsilon_{\alpha\beta}^* + 2K_{66}\varepsilon_{\alpha\beta} + k_2(K_{66}\varepsilon_{\alpha\beta}^* + 2D_{66}\varkappa_{\alpha\beta}) + (A_{16} + k_2E_{16})\frac{\partial\gamma_\alpha^{(0)}}{\partial\beta} + \\
&+(A_{26} + k_2E_{26})\gamma_\alpha^{(0)} + (B_{16} + k_2F_{16})\frac{\partial\gamma_\beta^{(0)}}{\partial\alpha} + (B_{26} + k_2F_{26})\gamma_\beta^{(0)}, \\
&N_{\beta\alpha} = C_{66}\varepsilon_{\alpha\beta}^* + 2K_{66}\varepsilon_{\alpha\beta} + k_1(K_{66}\varepsilon_{\alpha\beta}^* + 2D_{66}\varkappa_{\alpha\beta}) + (A_{16} + k_1E_{16})\frac{\partial\gamma_\alpha^{(0)}}{\partial\beta} + \\
&+(A_{26} + k_1E_{26})\gamma_\alpha^{(0)} + (B_{16} + k_1F_{16})\frac{\partial\gamma_\beta^{(0)}}{\partial\alpha} + (B_{26} + k_1F_{26})\gamma_\beta^{(0)}, \qquad (8) \\
&M_\alpha = K_{11}\varepsilon_{\alpha\alpha} + K_{12}\varepsilon_{\beta\beta} + D_{11}\varkappa_\alpha + D_{12}\varkappa_\beta + E_{11}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha} + E_{12}\gamma_\alpha^{(0)} + \\
&+F_{11}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + F_{12}\gamma_\beta^{(0)}, \quad M_\beta = K_{12}\varepsilon_{\alpha\alpha} + K_{22}\varepsilon_{\beta\beta} + D_{12}\varkappa_\alpha + D_{22}\varkappa_\beta + \\
&+E_{21}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha} + E_{22}\gamma_\alpha^{(0)} + F_{21}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + F_{22}\gamma_\beta^{(0)}, \quad Q_\alpha = K_1\gamma_\alpha^{(0)}, \quad Q_\beta = K_2\gamma_\beta^{(0)}, \\
&M_{\alpha\beta} = M_{\beta\alpha} = K_{66}\varepsilon_{\alpha\beta}^* + 2D_{66}\varkappa_{\alpha\beta} + E_{16}\frac{\partial\gamma_\alpha^{(0)}}{\partial\beta} + E_{26}\gamma_\alpha^{(0)} + F_{16}\frac{\partial\gamma_\beta^{(0)}}{\partial\alpha} + F_{26}\gamma_\beta^{(0)},
\end{aligned}
$$

where N_α, N_β, $N_{\alpha\beta}$, $N_{\beta\alpha}$ are the tangential forces; Q_α, Q_β are intersecting forces; M_α M_β are bending moments; $M_{\alpha\beta}$, $M_{\beta\alpha}$ are twisting moments; C_{ij}, K_{ij}, D_{ij}, K_1, K_2 are rigidity characters, which are determined by elastic parameters of the layers and their thickness; and A_{11}, A_{12}, ... F_{16}, F_{26} are values, depending on the geometrical and mechanical parameters of the layers of the shell [2].

The equations of the equilibrium of the element of shell have the following form:

$$
\begin{aligned}
&\frac{\partial BN_\alpha}{\partial\alpha} + \frac{\partial AN_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}N_{\alpha\beta} - \frac{\partial B}{\partial\alpha}N_\beta + ABk_1Q_\alpha^* + ABq_1 = 0, \\
&\frac{\partial AN_\beta}{\partial\beta} + \frac{\partial BN_{\alpha\beta}}{\partial\alpha} + \frac{\partial B}{\partial\alpha}N_{\beta\alpha} - \frac{\partial A}{\partial\beta}N_\alpha + ABk_2Q_\beta^* + ABq_2 = 0, \\
&\frac{\partial BQ_\alpha^*}{\partial\alpha} + \frac{\partial AQ_\beta^*}{\partial\beta} - ABk_1N_\alpha - ABk_2N_\beta + ABq_3 = 0, \\
&\frac{\partial BM_\alpha}{\partial\alpha} + \frac{\partial AM_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}M_{\alpha\beta} - \frac{\partial B}{\partial\alpha}M_\beta - ABQ_\alpha = 0,
\end{aligned}
\qquad (9)
$$

$$\frac{\partial AM_\beta}{\partial\beta}+\frac{\partial BM_{\alpha\beta}}{\partial\alpha}+\frac{\partial B}{\partial\alpha}M_{\beta\alpha}-\frac{\partial A}{\partial\beta}M_\alpha-ABQ_\beta=0,$$

where

$$\begin{aligned}Q^*_\alpha&=Q_\alpha-(N_\alpha+k_1M_\alpha)\theta_\alpha-(N_{\alpha\beta}+k_1M_{\alpha\beta})\theta_\beta,\\ Q^*_\beta&=Q_\beta-(N_{\beta\alpha}+k_2M_{\beta\alpha})\theta_\alpha-(N_\beta+k_2M_\beta)\theta_\beta.\end{aligned}\tag{10}$$

In the equations (9) the values $q_1,\ q_2\ q_3$ are the projections of the surface loaded in he direction of the coordinate lines $\alpha,\ \beta,\ \gamma$ respectively.

Let us consider the non -axially symmetric deformation of the laminated shells of rotation. We take the coordinate surface of the laminated shell of rotation before the deformation in the curvilinear orthogonal system $\alpha=s$, $\beta=\theta$ where s is the length of the arc of generator; θ is the central corner in the parallel disk. Starting from the fact that the functions $N_s,\ N_{s\theta},\ Q^*_s,\ M_s,\ M_{s\theta}$, $u,\ v,\omega,\ \Psi_s,\ \Psi_\theta$ are taken as solving, from the general equations we obtain the solving system of differential equations in the form

$$\begin{aligned}&\frac{\partial N_s}{\partial s}=-\frac{1}{r}\frac{\partial N_{s\theta}}{\partial\theta}-\frac{1}{r}(k_1-k_2)\frac{\partial M_{s\theta}}{\partial\theta}-\frac{cos\varphi}{r}(N_\theta-N_s)-k_1Q^*_s-q_1,\\ &\frac{\partial N_{s\theta}}{\partial s}=-\frac{1}{r}\frac{\partial N_\theta}{\partial\theta}--\frac{2cos\varphi}{r}N_{s\theta}-\frac{cos\varphi}{r}(k_1-k_2)M_{s\theta}-k_2Q^*_\theta-q_2,\\ &\frac{\partial Q^*_s}{\partial s}=-\frac{cos\varphi}{r}Q^*_s-\frac{1}{r}\frac{\partial Q^*_\theta}{\partial\theta}+k_1N_s+k_2N_\theta-q_3,\\ &\frac{\partial M_s}{\partial s}=-\frac{1}{r}\frac{\partial_{s\theta}}{\partial\theta}+\frac{cos\varphi}{r}(M_\theta-M_s)+Q^*_s+(N_s+k_1M_s)\Psi_s+(N_{s\theta}+k_1M_{s\theta})\Psi_\theta,\\ &\frac{\partial M_{s\theta}}{\partial s}=-2\frac{cos\varphi}{r}M_{s\theta}-\frac{1}{r}\frac{\partial M_\theta}{\partial\theta}-K_1\left(\Psi_\theta+\frac{1}{r}\frac{\partial\omega}{\partial\theta}-k_2v\right),\ \frac{\partial u}{\partial s}=\varepsilon_s-k_1\omega,\\ &\frac{\partial v}{\partial s}=\varepsilon_{s\theta}-\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{cos\varphi}{r}v,\ \ \frac{\partial\omega}{\partial s}=k_1u+\frac{1}{r}\frac{\partial\omega}{\partial\theta}-k_2v,\ \ \frac{\partial\Psi_s}{\partial s}=k_1\varepsilon_s+\varkappa_s,\\ &\frac{\partial\Psi_\theta}{\partial s}=2\varkappa_{s\theta}-\frac{1}{r}\frac{\partial\Psi_s}{\partial\theta}+\frac{cos\varphi}{r}\Psi_\theta+k_1\varepsilon_{s\theta}+(k_2-k_1)\left(\frac{1}{r}\frac{\partial u}{\partial\theta}-\frac{cos\varphi}{r}\right)v.\end{aligned}\tag{11}$$

The values $N_\theta,\ Q^*_\theta,\ M_\theta,\ \varepsilon_s,\ \varepsilon_{s\theta},\ \varkappa_s,\ \varkappa_{s\theta}$ involved in the equations (11) have the following form

$$\begin{aligned}&\varepsilon_s=b_{11}N_s+b_{12}M_s+b_{13}\varepsilon_{\theta\theta}+b_{14}\varepsilon_\theta+b_{15}\frac{\partial\gamma_s^{(0)}}{\partial s}+b_{16}\gamma_s^{(0)}+b_{17}\frac{\partial\gamma_\theta^{(0)}}{\partial\theta}+\\ &+b_{18}\gamma_\theta^{(0)}-\frac{1}{2}(\Psi_s-\gamma_s^{(0)})^2,\\ &Q^*_\theta=K_2\left(\Psi_\theta+\frac{1}{r}\frac{\partial\omega}{\partial\theta}-k_2\omega\right)-(N_{s\theta}+k_1M_{s\theta})\Psi_s-(N_\theta+k_2M_\theta)\Psi_\theta,\\ &\varkappa_s=b_{21}N_s+b_{22}M_s+b_{23}\varepsilon_{\theta\theta}+b_{24}\varkappa_\theta+b_{25}\frac{\partial\gamma_s^{(0)}}{\partial s}+b_{26}\gamma_s^{(0)}+b_{27}\frac{\partial\gamma_\theta^{(0)}}{\partial\theta}+b_{28}\gamma_\theta^{(0)},\\ &N_\theta=b_{31}N_s+b_{32}M_s+b_{33}\varepsilon_{\theta\theta}+b_{34}\varkappa_\theta+b_{35}\frac{\partial\gamma_s^{(0)}}{\partial s}+b_{36}\gamma_s^{(0)}+b_{37}\frac{\partial\gamma_\theta^{(0)}}{\partial\theta}+b_{38}\gamma_\theta^{(0)},\\ &M_\theta=b_{41}N_s+b_{42}M_s+b_{43}\varepsilon_{\theta\theta}+b_{44}\varkappa_\theta+b_{45}\frac{\partial\gamma_s^{(0)}}{\partial s}+b_{46}\gamma_s^{(0)}+b_{47}\frac{\partial\gamma_\theta^{(0)}}{\partial\theta}+b_{48}\gamma_\theta^{(0)},\end{aligned}$$

$$
\begin{aligned}
&\varepsilon_{s\theta} = b_{51}N_{s\theta} + b_{52}M_{s\theta} + b_{53}\tfrac{\partial\gamma_s^{(0)}}{\partial\theta} + +b_{54}\gamma_s^{(0)} + b_{55}\tfrac{\partial\gamma_\theta^{(0)}}{\partial s} + b_{56}\gamma_\theta^{(0)} - \\
&-(\Psi_s - \gamma_s^{(0)})(\Psi_\theta - \gamma_\theta^{(0)}), \\
&2\varkappa_{s\theta} = b_{61}N_{s\theta} + b_{62}M_{s\theta} + b_{63}\frac{\partial\gamma_s^{(0)}}{\partial\theta} + b_{64}\gamma_s^{(0)} + b_{65}\frac{\partial\gamma_s^{(0)}}{\partial s} + b_{66}\gamma_s^{(0)}.
\end{aligned} \tag{12}
$$

In the expressions (12) the values $\varepsilon_{\theta\theta},\ \varkappa_\theta,\ \gamma_s^{(0)},\ \gamma_\theta^{(0)},\ \frac{\partial\gamma_s^{(0)}}{\partial s},\ \frac{\partial\gamma_s^{(0)}}{\partial\theta},\ \frac{\partial\gamma_\theta^{(0)}}{\partial s},\ \frac{\partial\gamma_\theta^{(0)}}{\partial\theta}$ are expressed by the solving functions in the following form

$$
\varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{cos\varphi}{r}u + k_2 w + \frac{1}{2}\left(k_2 v - \frac{1}{r}\frac{\partial w}{\partial\theta}\right)^2,
$$

$$
\varkappa_\theta = \frac{1}{r}\frac{\partial\Psi_\theta}{\partial\theta} + \frac{cos\varphi}{r}\Psi_s - k_2\left(\frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{cos\varphi}{r}u + k_2 w\right),
$$

$$
\gamma_s^{(0)} = K_1^{-1}\left[Q_s^* + (N_s + k_1 M_s)\Psi_s + (N_{s\theta} + k_1 M_{s\theta})\Psi_\theta\right],
$$

$$
\begin{aligned}
&\gamma_\theta^{(0)} = \Psi_\theta + \frac{1}{r}\frac{\partial w}{\partial\theta} - k_2 w, \\
&\frac{\partial\gamma_s^{(0)}}{\partial\theta} = K_1^{-1}\left[\frac{\partial Q_s^*}{\partial\theta} + \left(\frac{\partial N_s}{\partial\theta} + k_1\frac{\partial M_s}{\partial\theta}\right)\Psi_s + \left(\frac{\partial N_{s\theta}}{\partial\theta} + \right.\right. \\
&\left.\left. + k_1\frac{\partial M_{s\theta}}{\partial\theta}\right)\Psi_\theta + (N_s + k_1 M_s)\frac{\partial\Psi_s}{\partial\theta} + (N_{s\theta} + k_1 M_{s\theta})\frac{\partial\Psi_\theta}{\partial\theta}\right], \\
&\frac{\partial\gamma_\theta^{(0)}}{\partial\theta} = \frac{\partial\Psi_\theta}{\partial\theta} + \frac{1}{r}\frac{\partial^2 w}{\partial\theta^2} + k_2\frac{\partial v}{\partial\theta},
\end{aligned} \tag{13}
$$

$$
\frac{\partial\gamma_\theta^{(0)}}{\partial s} = b_1 N_{s\theta} + b_2 M_{s\theta} + b_3\frac{\partial\gamma_s^{(0)}}{\partial\theta} + b_4\gamma_s^{(0)} + b_5\gamma_\theta^{(0)} +
$$

$$
+ b_6(\Psi_s - \gamma_s^{(0)})(\Psi_\theta - \gamma_\theta^{(0)}) + b_7\frac{\partial\Psi_s}{\partial\theta} + b_8\frac{\partial u}{\partial\theta} + b_9\frac{\partial w}{\partial\theta} + b_{10}\Psi_\theta + b_{11}^* u + b_{12}^* v,
$$

$$
\frac{\partial\gamma_s^{(0)}}{\partial s} = \frac{1}{c_0 - c_1 N_s - c_2 M_s - c_3\Psi_s - c_4\Psi_\theta - \frac{\partial\Psi_\theta}{\partial\theta}}(a_1 N_s + a_2 Q_s^* + a_3 M_s + a_4 u +
$$

$$
+ a_5 v + a_6 w + a_7\Psi_s + a_8\Psi_\theta + a_9 N_{s\theta}\Psi_s + a_{10}M_{s\theta}\Psi_s + a_{11}M_s\Psi_s + a_{12}N_s\Psi_\theta)
$$

The coefficients $b_{11},\ b_{12},\ ...,\ a_{11},\ a_{12}$ involved in the expressions (12), (13) are determined by geometrical and mechanical parameters of the layers of the shells.

Adding to the equations (11) the corresponding boundary conditions, we obtain the nonlinear boundary problem. At this the boundary conditions are given in the following way: the static ones - by forces and the moment - in the integral form, the kinematic ones in discrete number of points of the end-wall of the shell.

References

[1] Grigorenko Y. M., Vasilenko A.T., On Considering the Nonhomogeneity of Deformations of the Transverse Shear in the Thickness in the Laminated Shells. //Appl. Mechanics, 1977, 13, N 10, pp. 36-42.

[2] Grigorenko Y.M., Vasilenko A.T., The Theory of Shells of Variable Rigidity. Kiev: Naukova Dumka, 1981, 544 p.

[3] Novojilov V.V., The Basis of Nonlinear Theory of Elasticity. M.: Gostexizdat, 1948, 212 p.

[4] Shapovalov L.A., On One Simple Variant of Equations of Geometric Nonlinear Theory of Thin Shells.// Ing. jur. MTT. 1968, N 1, pp. 56-62.