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Correction to "Open books and configurations of symplectic surfaces"

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Abstract We correct the main theorem in "Open books and configurations of symplectic surfaces" [1] and its proof. As originally stated, the theorem gave conditions on a configuration of symplectic surfaces in a symplectic 4-manifold under which we could construct a model neighborhood with concave boundary and describe explicitly the open book supporting the contact structure on the boundary. The statement should have included constraints on the areas of the surfaces.

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In the paper being corrected [1], we considered symplectic configuration graphs where each vertex was decorated with a triple (g_i, m_i, a_i), g_i being the genus of a surface, m_i its self-intersection, and a_i its symplectic area. Theorem 1.1 in [1] is false as stated. However we have the following:

Correction 1 *The conclusions of theorem 1.1 (parts A and B) are true if we add the hypothesis that there exists a constant $\lambda > 0$ such that, for each i, $a_i = \lambda(m_i + d_i)$.*

An easy counterexample to the theorem as originally stated is given by two surfaces Σ_1 and Σ_2, with $\Sigma_1 \cdot \Sigma_1 = \Sigma_2 \cdot \Sigma_2 = 1$ and $\Sigma_1 \cdot \Sigma_2 = 1$. If $\int_{\Sigma_1} \omega = a_1 \neq \int_{\Sigma_2} \omega = a_2$, then $[\omega] \neq 0$ when restricted to the boundary of a neighborhood of $\Sigma_1 \cup \Sigma_2$, because $[\omega]$ is nonzero when evaluated on the class $[\Sigma_1] - [\Sigma_2]$, which lives on the boundary. In general, this problem occurs when the intersection matrix for the configuration of surfaces has determinant equal to zero.

The error is on line 4 of page 583 in [1], in the calculation of α_H^-. The correct statement is:

$$\alpha_H^- = kc_i(d\mu + d\lambda) - \alpha_H^+$$

Thus, to arrange that this agree with the contact pair we already have on a neighborhood of $K \subset \partial X$, we are forced to choose $c_i = 1$, as opposed to

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$c_i = a_i/(2\pi K(m_i + d_i))$, which was the choice made on page 582 line 22. With $c_i = 1$, we get that the area of Σ_i is $2\pi(m_i + d_i)k$. After the construction we can rescale by a constant to get the area to be $\lambda(m_i + d_i)$.

Note that, if the configuration $\Sigma_1 \cup \ldots \cup \Sigma_n$ is contained in a closed symplectic 4-manifold (X, ω), then the area condition added in this correction is equivalent to the condition that $[\omega]$ is Poincaré dual to some multiple of $[\Sigma_1]+\ldots+[\Sigma_n]+\gamma$, where $\gamma \in H_2(X;\mathbb{Z})$ is a class with $\gamma \cdot \Sigma_i = 0$ for $i = 1, \ldots, n$. This is because $m_i + d_i = \Sigma_1 \cdot \Sigma_i + \ldots + \Sigma_n \cdot \Sigma_i$.

Here we emphasize that this correction does not affect the applications in section 2 of [1], or forthcoming applications in [2].

References

[1] **D T Gay** *Open books and configurations of symplectic surfaces*, Alg. Geom. Top. 3 (2003), 569–586

[2] **D T Gay and R Kirby** *Constructing symplectic forms on 4-manifolds which vanish on circles*, to appear

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