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Lefschetz fibrations on compact Stein surfaces

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Abstract

Let M be a compact Stein surface with boundary. We show that M admits infinitely many pairwise nonequivalent positive allowable Lefschetz fibrations over D^2 with bounded fibers.

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0 Introduction

The existence of a positive allowable Lefschetz fibration on a compact Stein surface with boundary was established by Loi and Piergallini [10] using branched covering techniques. We give an alternative simple proof of this fact and construct explicitly the vanishing cycles of the Lefschetz fibration, obtaining a direct identification of compact Stein surfaces with positive allowable Lefschetz fibrations over D^2. In the process we associate to every compact Stein surface infinitely many pairwise nonequivalent such Lefschetz fibrations.

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1 Preliminaries

1.1 Mapping class groups

Let F be a compact, oriented and connected surface with boundary. Let $\mathrm{Diff}^+(F, \partial F)$ be the group of all orientation preserving self diffeomorphisms of F, fixing boundary pointwise and let $\mathrm{Diff}_0^+(F, \partial F)$ be the subgroup of $\mathrm{Diff}^+(F, \partial F)$ consisting of all self diffeomorphisms isotopic to the identity. Then we define the *mapping class group* of the surface F as

$$Map(F, \partial F) = \mathrm{Diff}^+(F, \partial F)/ \mathrm{Diff}_0^+(F, \partial F).$$

A positive (or right-handed) *Dehn twist* $D(\alpha)\colon F \to F$ about a simple closed curve α is a diffeomorphism obtained by cutting F along α, twisting 360° to the right and regluing. Note that the positive Dehn twist $D(\alpha)$ is determined up to isotopy by α and is independent of the orientation on α.

It is well-known that the mapping class group $Map(F, \partial F)$ is generated by Dehn twists. We will use the functional notation for the products in $Map(F, \partial F)$, eg, $D(\beta)D(\alpha)$ will denote the composition where we apply $D(\alpha)$ first and then $D(\beta)$.

1.2 Surface bundles over circle

In this paper we use the following convention for the monodromy of a surface bundle over a circle. We say that an F–bundle W over S^1 has monodromy h iff W is diffeomorphic to

$$(F \times I)/\ (h(x), 0) \sim (x, 1)$$

where $h \in Map(F, \partial F)$. In other words, h is the monodromy if we travel around the base circle in the positive normal direction to the surface F. Consider the closed 3–manifold

$$W' = W \cup_{\partial} (\partial F \times D^2).$$

We say that W' has an *open book decomposition* with binding ∂F, page F and monodromy h. It is well-known that every closed 3–manifold admits an open book decomposition.

1.3 Positive Lefschetz fibrations

Let M be a compact, oriented smooth 4–manifold, and let B be a compact, oriented smooth 2–manifold. A smooth map $f\colon M \to B$ is a *positive Lefschetz fibration* if there exist points $b_1, \dots, b_m \in$ *interior* (B) such that

(1) $\{b_1, \dots, b_m\}$ are the critical values of f, with $p_i \in f^{-1}(b_i)$ a unique critical point of f, for each i, and

(2) about each b_i and p_i, there are local complex coordinate charts agreeing with the orientations of M and B such that locally f can be expressed as $f(z_1, z_2) = z_1^2 + z_2^2$.

It is a consequence of this definition that

$$f|_{f^{-1}(B - \{b_1, \dots, b_m\})}\colon f^{-1}(B - \{b_1, \dots, b_m\}) \to B - \{b_1, \dots, b_m\}$$

is a smooth fiber bundle over $B - \{b_1, \dots, b_m\}$ with fiber diffeomorphic to an oriented surface F.

Two positive Lefschetz fibrations $f\colon M \to B$ and $f'\colon M' \to B'$ are *equivalent* if there are diffeomorphisms $\Phi\colon M \to M'$ and $\phi\colon B \to B'$ such that $f'\Phi = \phi f$.

If $f\colon M \to D^2$ is a positive Lefschetz fibration, then we can use this fibration to produce a handlebody description of M. We select a regular value $b_0 \in$ *interior* (D^2) of f, an identification $f^{-1}(b_0) \cong F$, and a collection of arcs s_i in *interior* (D^2) with each s_i connecting b_0 to b_i, and otherwise disjoint from the other arcs. We also assume that the critical values are indexed so that the arcs $s_1, \ldots, s_m$ appear in order as we travel counterclockwise in a small circle about b_0. Let $V_0, \ldots, V_m$ denote a collection of small disjoint open disks with $b_i \in V_i$ for each i. (cf Figure 1).

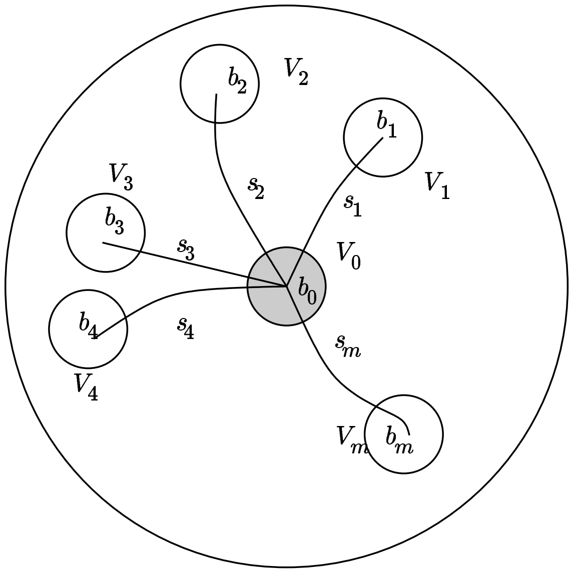

Figure 1: Fibration over the disk

To build our description of M, we observe first that $f^{-1}(V_0) \cong F \times D^2$, with $\partial(f^{-1}(V_0)) \cong F \times S^1$. Let $\nu(s_i)$ be a regular neighborhood of the arc s_i. Enlarging V_0 to include the critical value b_1, it can be shown that $f^{-1}(V_0 \cup \nu(s_1) \cup V_1)$ is diffeomorphic to $F \times D^2$ with a 2–handle H_1 attached along a circle γ_1 contained in a fiber $F \times pt \subset F \times S^1$. Moreover, condition (2) in the definition of a Lefschetz fibration requires that H_1 is attached with a framing -1 relative to the natural framing on γ_1 inherited from the product structure of $\partial(f^{-1}(V_0))$. γ_1 is called a vanishing cycle. In addition, $\partial((F \times D^2) \cup H_1)$ is diffeomorphic to an F–bundle over S^1 whose monodromy is given by $D(\gamma_1)$, a positive Dehn twist about γ_1. Continuing counterclockwise about b_0, we add the remaining critical values to our description, yielding that

$$M_0 \cong f^{-1}(V_0 \cup (\bigcup_{i=1}^{m} \nu(s_i)) \cup (\bigcup_{i=1}^{m} V_i))$$

is diffeomorphic to $(F \times D^2) \cup (\bigcup_{i=1}^{m} H_i)$, where each H_i is a 2–handle attached along a vanishing cycle γ_i in an F–fiber in $F \times S^1$ with relative framing -1. (For a proof of these statements see [9] or [6].)

Furthermore,

$$\partial M_0 \cong \partial((F \times D^2) \cup (\bigcup_{i=1}^{m} H_i))$$

is an F–bundle over S^1 with monodromy given by the composition $D(\gamma_m) \cdots D(\gamma_1)$. We will refer to this product $D(\gamma_m) \cdots D(\gamma_1)$ as the *global monodromy* of this fibration.

We note that we can reverse this argument to construct a positive Lefschetz fibration over D^2 from a given set of vanishing cycles.

We say that a positive Lefschetz fibration is *allowable* iff all its vanishing cycles are homologically non-trivial in the fiber F. Note that a simple closed curve on a surface is homologically trivial iff it separates the surface.

Notation *PALF* means a positive allowable Lefschetz fibration over D^2 with bounded fibers.

Remark 1 With this new notation, we can summarize the handle attaching procedure as

$$PALF \ \cup \ \text{Lefschetz 2–handle} = PALF$$

where a Lefschetz 2–handle is a 2–handle attached along a nonseparating simple closed curve in the boundary with framing -1 relative to the product framing.

1.4 Contact structures

We use the standard tight contact structures on $\mathbb{R}^3$, S^3 and $\#_n S^1 \times S^2$ (for $n \geq 1$) compatible with their standard orientations. The structures on S^3 and $\#_n S^1 \times S^2$ are uniquely (up to blowups) holomorphically fillable — S^3 as the boundary of $D^4 \subset \mathbb{C}^2$ and $\#_n S^1 \times S^2$ as the boundary of D^4 union n 1–handles. The tight contact structure on $\mathbb{R}^3 \subset S^3$ will be represented by the kernel of the 1–form $dz + xdy$.

A link L in a contact manifold is called *Legendrian* if its tangent vectors all lie in the contact planes. Legendrian link theory in $\mathbb{R}^3$ or S^3 reduces to the theory of the corresponding front projections in $\mathbb{R}^2$. We will use projections onto the yz–plane in this paper. The Thurston–Bennequin invariant of a Legendrian knot L, denoted by $tb(L)$, can be computed from a front projection diagram of L as

$$bb(L) - \#\text{left cusps}$$

where $bb(L)$ is the blackboard framing of L.

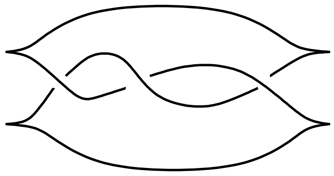

Figure 2: Legendrian trefoil knot

2 Torus knots

Let p and q be relatively prime integers such that $p, q \geq 2$.

Theorem 1 *The monodromy of a (p, q) torus knot is a product of $(p-1)(q-1)$ nonseparating positive Dehn twists.*

Proof It is well-known that a torus knot is fibered with fiber being its minimal Seifert surface. We will describe how to construct this fiber by plumbing left-handed Hopf bands (cf [7]).

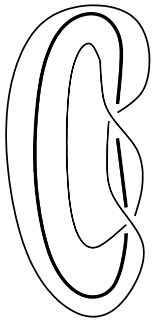

Figure 3: Left-handed Hopf band

The monodromy of a left-handed Hopf band is a positive Dehn twist along its core circle as shown in Figure 3. Note that our convention for monodromy (see section 1.2) differs from Harer's in [7].

It is proven in [16] (see also [4]) that the monodromy of a surface obtained by plumbing two surfaces is the composition of their monodromies. We can plumb two left-handed Hopf bands to get a $(2, 3)$ torus knot with its fibered surface. Simply identify a neigborhood of the arc α in one Hopf band with a neighborhood of the arc β in the other Hopf band, transversally as shown in

Figure 4. The resulting monodromy will be the product of two positive Dehn twists along the curves also drawn in Figure 4. Note that the two curves (one of which is drawn thicker) intersect each other only once and they stay parallel when they go through the left twist on the surface. It is clear that we can iterate this plumbing operation to express the monodromy of a $(2,q)$ torus knot as a product of $(q-1)$ positive Dehn twists.

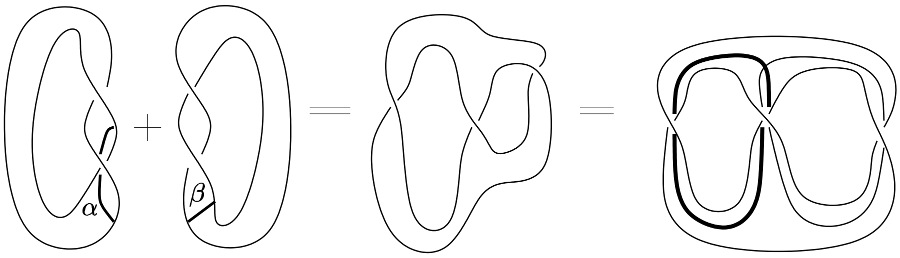

Figure 4: Plumbing two Hopf bands

By attaching more left-handed Hopf bands we can construct the fibered surface of a (p,q) torus knot for arbitrary p and q. First construct the gate in the back and then plumb a Hopf band in the front face of that gate and proceed as above to obtain a second gate. We can iterate this process to get as many gates as we want. This is illustrated for $p=3$ and $q=5$ in Figure 5.

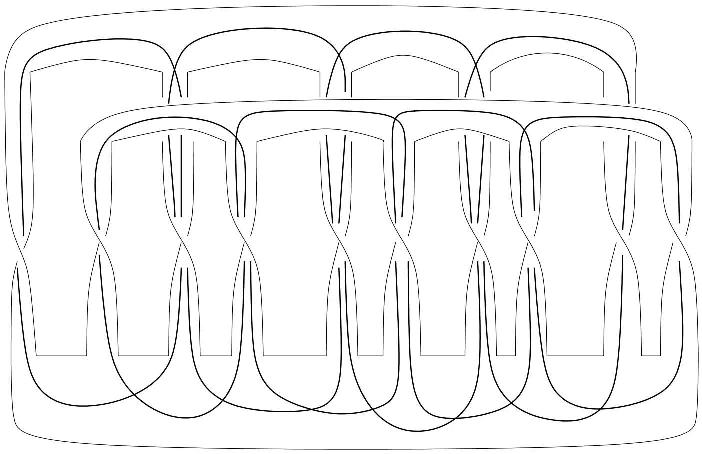

Figure 5: Monodromy of a $(3,5)$ torus knot

Hence the monodromy of the fibration of the complement of a torus knot in S^3 is a product of positive Dehn twists. These twists are nonseparating by our construction. □

Remark 2 Our construction also shows that the monodromy of an arbitrary torus link is a product of positive Dehn twists.

Theorem 2 [11] *Let L be a link in S^3. There exists a torus knot $K \subset S^3$ such that $K \cap L = \emptyset$ and $L \subset F$ where F is a minimal Seifert surface for K. Moreover no component of L separate the surface F.*

Proof We describe Lyon's construction given in [11]. We say that a link in $\mathbb{R}^3$ is in a *square bridge position* with respect to the plane $x = 0$ if the projection onto the plane is regular and each segment above the plane projects to a horizontal segment and each one below to a vertical segment. Clearly any link can be put in a square bridge position.

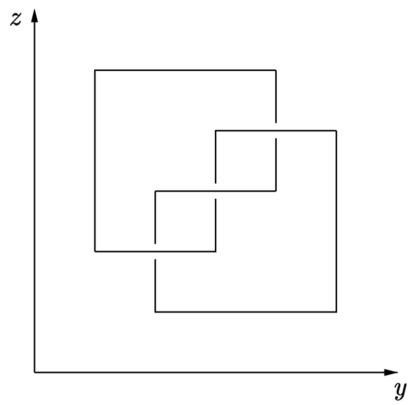

Figure 6: Trefoil knot in a square bridge position

Suppose that the horizontal and vertical segments of the projection of the link in the yz–plane are arranged by isotopy so that each horizontal segment is a subset of

$$\{0\} \times [0,1] \times \{z_i\}$$

for some $0 < z_1 < z_2 < \ldots < z_p < 1$ and and each vertical segment is a subset of

$$\{0\} \times \{y_j\} \times [0,1]$$

for some $0 < y_1 < y_2 < \ldots < y_q < 1$. Now consider the 2–disk

$$D_i = [\epsilon, 1] \times [0, 1] \times \{z_i\}$$

for each $i = 1, 2, \ldots, p$ and the 2–disk

$$E_j = [-1, -\epsilon] \times \{y_j\} \times [0, 1]$$

for each $j = 1, 2, \ldots, q$, where ϵ is a small positive number. Attach these disks by small bands (see Figure 7) corresponding to each point $(0, y_i, z_j)$ for $i = 1, \ldots, p$ and $j = 1, \ldots q$. If p and q are relatively prime then the result is the minimal Seifert surface F for a (p, q) torus knot K such that $K \cap L = \emptyset$ and $L \subset F$. Each component of the link L is a nonseparating embedded curve on the surface F since we can find an arc connecting that component to the boundary K from either side of the component. Moreover we can choose p and q arbitrarily large by adding more disks of either type D or type E. □

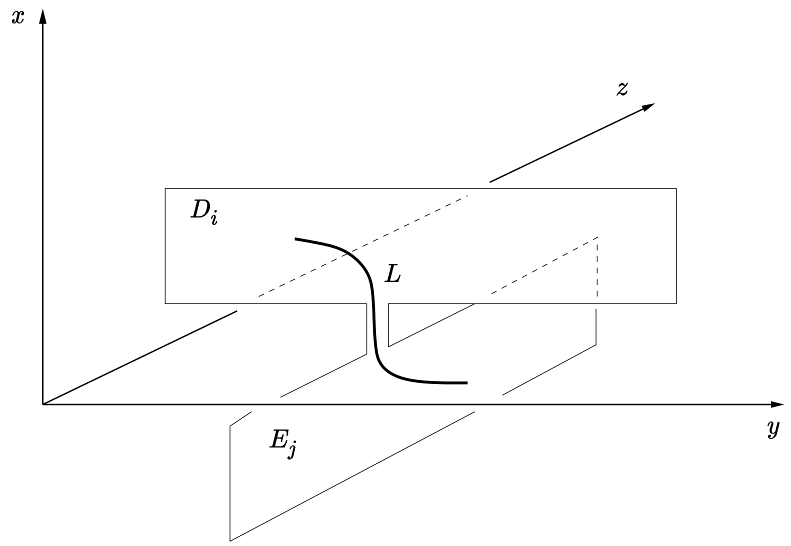

Figure 7: Attaching disks

3 Main theorems

Let K be a torus knot in S^3. Since K is a fibered knot, this gives an open book decomposition of S^3 with monodromy h which is a product $D(\gamma_m) \cdots D(\gamma_1)$ of nonseparating positive Dehn twists given by Theorem 1. Then S^3 bounds a

$(PALF)_K$ with global monodromy $D(\gamma_m)\cdots D(\gamma_1)$ and fiber F which is the minimal Seifert surface for K.

Proposition 3 *For any torus knot K, $(PALF)_K$ is diffeomorphic to D^4 and has a canonical Stein structure.*

Proof Consider the handle decomposition of the $(PALF)_K$ for a torus knot K. Theorem 1 gives an explicit description of the vanishing cycles. Cancel each 1–handle with a 2–handle so that the result is just the 0–handle D^4. □

Theorem 4 (Eliashberg [3], see also Gompf [5]) *A smooth oriented compact 4–manifold with boundary is a Stein surface, up to orientation preserving diffeomorphisms, iff it has a handle decomposition $M_1 \cup H_1 \cup \ldots \cup H_n$, where M_1 consists of 0– and 1–handles and each H_i is a 2–handle attached to M_1 along some attaching circle L_i with framing $tb(L_i) - 1$.*

We are now ready to state and prove our main theorem.

Theorem 5 *Let M be a compact Stein surface with boundary. Then M admits infinitely many pairwise nonequivalent PALF's. Conversely every PALF has a Stein structure.*

Proof Let M be a compact Stein surface with boundary. We use Eliashberg's characterization of compact Stein surfaces.

Case 1 : no 1–handles and one 2–handle

Suppose that the compact Stein surface M with boundary is obtained by attaching a 2–handle H to D^4 along a Legendrian knot L, with framing $tb(L)-1$. Figure 2 shows the front projection of a Legendrian trefoil knot. First of all, we smooth all the cusps of the diagram and rotate everything counterclockwise to put L into a square bridge position as in Figure 6.

Now we use Lyon's algorithm (cf Theorem 2) to find a torus knot K with its minimal Seifert surface F such that L is an embedded circle on the surface F. For example, we can embed the trefoil knot into the Seifert surface of a (5,6) torus knot as shown in Figure 8. Let L^+ be a copy of L pushed in the positive normal direction to F, and let $lk(L,L^+)$ be the linking number of L and L^+ computed with parallel orientations. We need the following observation to prove our theorem.

Lemma 6 $tb(L) = lk(L, L^+)$.

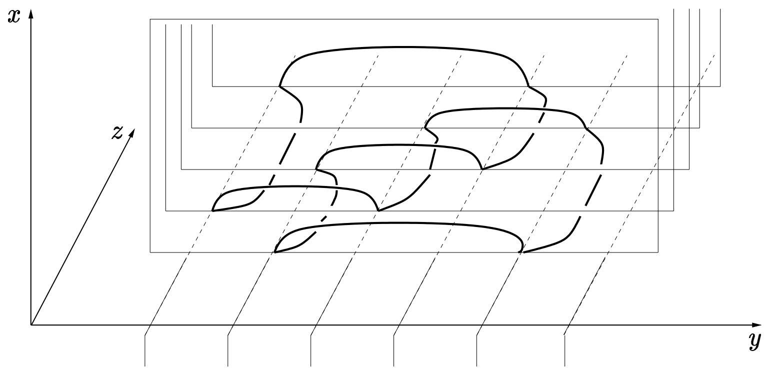

Figure 8: Trefoil knot embedded into the Seifert surface of a $(5,6)$ torus knot

Proof When we push L in the positive normal direction to F, we observe that $lk(L, L^+)$ will be exactly the Thurston–Bennequin framing of L, by simply counting the linking number of L and L^+. □

Therefore attaching a 2–handle to D^4 along a given Legendrian knot L in S^3, with framing $tb(L) - 1$, is the same as attaching a 2–handle along the same knot L (which is isotoped to be embedded in a fiber of the boundary of a $(PALF)_K$) with framing $lk(L, L^+) - 1$. But then the framing $lk(L, L^+) - 1$ is the framing -1 relative to the product framing of L. In other words, we proved that attaching a Legendrian 2–handle is the same as attaching a Lefschetz 2–handle in our setting.

The global monodromy of $D^4 \cup H \cong (PALF)_K \cup H$ will be the monodromy of the torus knot K composed with a positive Dehn twist along L.

Case 2 : no 1–handles

Let L be a Legendrian link in S^3 with components $L_1, L_2, \ldots, L_n$. Suppose that the compact Stein surface M with boundary is obtained by attaching a 2–handle H_i to D^4 along L_i for each $i = 1, 2, \ldots, n$. First smooth all the cusps of the diagram and rotate everything counterclockwise to put L into a square bridge position. Then find a torus knot K with its minimal Seifert surface F such that each L_i is an embedded circle on F for $i = 1, 2, \ldots, n$. Now for each i, attach a 2–handle H_i simultaneously to D^4 along L_i with framing $lk(L_i, L_i^+) - 1$. The result is going to be a PALF by Lemma 6 and Remark 1, since the link components are disjointly embedded nonseparating circles in F.

So we showed the global monodromy of $D^4 \cup H_1 \cup \ldots \cup H_n \cong (PALF)_K \cup H_1 \cup \ldots \cup H_n$ is the monodromy of the torus knot K composed with positive Dehn twists along L_i's. Note that the Dehn twists along L_i's commute since they are pairwise disjoint on the surface F.

General case

First we represent the 1–handles by dotted circles stacked over the front projection of a Legendrian tangle. Here we assume that the framed link diagram is in standard form (cf [5]). Then we put the diagram in a square bridge position as in case 2 and find a torus knot K such that the attaching circles of the 2–handles are embedded in the Seifert surface of K. Now we push the attaching regions of the 1–handles to the binding K of the open book decomposition of S^3. This way we can extend $(PALF)_K$ to a PALF on D^4 union 1–handles. Since the attaching region (a pair of 3–balls) of the 1–handle is in a neighborhood of the binding K, we can assume that the pages of the open book will intersect the pair of balls transversally, so that afer gluing the 1–handle to D^4 we can extend the fibration over the 1–handle by adding a 1–handle to the surface of the fibration without altering the monodromy.

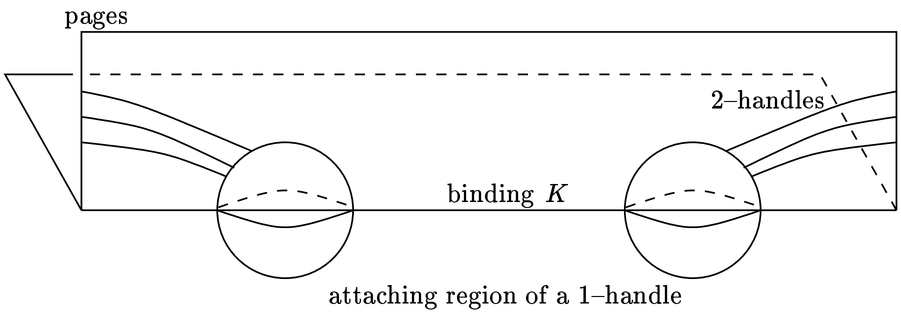

Figure 9: Attaching a 1–handle

Suppose that we attached all the 1–handles to D^4. Then case 2 implies that attaching a Legendrian 2–handle at this stage is the same as attaching a Lefschetz 2–handle, since neither the product framing nor the Thurston–Bennequin framing will change by going over a 1–handle. In other words, we showed that by attaching 1–handles to D^4 we attach 1–handles to the fiber of the $(PALF)_K$, extending the monodromy by identity over the 1–handles and then by attaching Legendrian 2–handles we add more vanishing cycles to the global monodromy.

Finally, we note that the (p, q) torus knot in Theorem 2 can be constructed using arbitrarily large p and q. Therefore our construction yields infinitely many pairwise nonequivalent PALF's, since for chosen p and q the genus of the regular fiber will be at least $(p-1)(q-1)/2$.

Conversely, let X be a PALF, then it is obtained by a sequence of steps of attaching 2–handles $X_0 = D^2 \times F \rightsquigarrow X_1 \rightsquigarrow X_2 .. \rightsquigarrow X_n = X$, where each X_{i-1} is a PALF and X_i is obtained from X_{i-1} by attaching a 2–handle to a nonseparating curve C lying on a fiber $F \subset \partial X_{i-1}$. Furthermore this handle is attached to C with the framing $k-1$, where k is the framing induced from the surface F. Inductively we assume that X_{i-1} has a Stein structure, with a convex fiber $F \subset \partial X_{i-1}$. By [17] we can start the induction, and assume that the convex surface F is divided by ∂F. By the "Legendrian realization principle" of [8] (pages 323–325), after an isotopy of (F, C), k can be taken to be the Thurston–Bennequin framing, and then the result follows by Eliashberg's theorem (L. Rudolph has pointed out that, in case of $i = 1$ identification of k with Thurston–Bennequin framing also follows from [12]–[15]). Though not necessary, in this process, by using [8] we can also make the framing of ∂F induced from F to be the Thurston–Bennequin framing if we wish. □

Remark 3 Our proof shows that the PALF structure on a compact Stein surface contains a natural smaller PALF $B^4 \to D^2$ given by the associated torus knot.

Remark 4 Our proof shows that by relaxing the condition of positivity, one can identify smooth bounded 4–manifolds which are built by 1– and 2–handles with ALF's (allowable Leschetz fibrations over D^2's). In this case in the proof we start with the binding $K \sharp (-K)$ where K is the torus knot, to adjust the framings (ie, we use the general form of [11]). In particular by [17], the boundaries of these manifolds also have contact structures (though not necessarily tight).

4 Examples

4.1 Example 1

Let M be the Stein surface given as in Figure 10.

We claim that M admits a PALF with 13 singular fibers where the regular fiber is a genus 6 surface with 2 boundary components.

First we embed the attaching curve of the 2–handle into the Seifert surface of a torus knot, such that the top two horizontal lines (running over the 1–handle) in

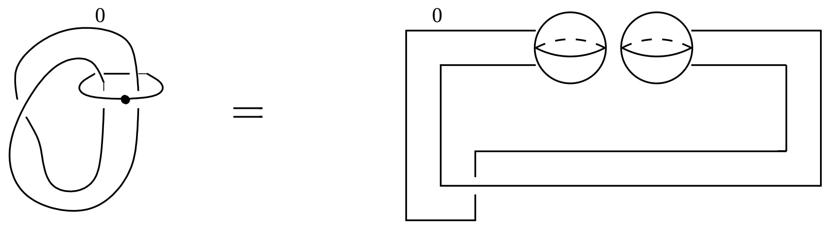

Figure 10

Figure 10 are placed on the same disk of type D (cf Theorem 2 and Figure 9). So we attach 4 disks of type D and 5 disks of type E to obtain the Seifert surface of a $(4,5)$ torus knot. By attaching the 1–handle we attach a 1–handle to the Seifert surface. Hence the fiber is a genus $6=(4-1)(5-1)/2$ surface with two boundary components. There are $12=(4-1)(5-1)$ singular fibers coming from the torus knot and one additional singular fiber corresponding to the 0–framed 2–handle.

The global monodromy of this PALF is the monodromy of the $(4,5)$ torus knot, extended by identity over the 1–handle and composed with a Dehn twist corresponding to the 2–handle.

4.2 Example 2

Let N be the Stein surface given as in Figure 11. Then N admits a PALF with 43 singular fibers where the regular fiber is a genus 21 surface with 2 boundary components. The global monodromy of this PALF is the monodromy of the $(7,8)$ torus knot, extended by identity over the 1–handle and composed with a Dehn twist corresponding to the 2–handle.

Remark 5 Clearly the PALF's in the above examples are not the most economical ones; often with a little care one can find smaller PALF structures in the sense of having fewer singular fibers.

Remark 6 In [1] it was shown that every smooth closed 4–manifold X can be decomposed as a union of two compact Stein surfaces along their boundaries

$$X=M\cup_\partial N.$$

Hence, every X is a union two PALF's along their boundaries. This gives 4–manifolds a structure somewhat similar to Heegaard decomposition of 3 manifolds (we can consider a 3–dimensional solid handlebody as a Lefschetz fibration

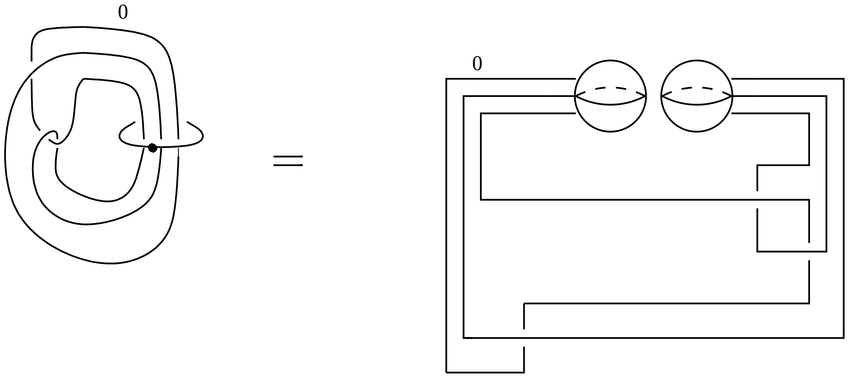

Figure 11

over an interval, with fibers consisting of disks). Recall that in [1] there is also a relative version of this theorem; that is, any two smooth closed simply connected h–cobordant manifolds X_1, X_2 can be decomposed as union of Stein surfaces $X_i = M \cup_{\varphi_i} W_i$, where $\varphi_i\colon \partial W_i \to \partial M$ are diffeomorphisms $i = 1, 2$, M is simply connected, and W_1, W_2 are contractible manifolds which are diffeomorphic to each other. See also [2] for more about the topology of Stein surfaces.

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