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On Invariants of Hirzebruch and Cheeger–Gromov

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Abstract

We prove that, if M is a compact oriented manifold of dimension $4k+3$, where $k>0$, such that $\pi_1(M)$ is not torsion-free, then there are infinitely many manifolds that are homotopic equivalent to M but not homeomorphic to it. To show the infinite size of the structure set of M, we construct a secondary invariant $\tau_{(2)}\colon S(M) \to \mathbb{R}$ that coincides with the ρ–invariant of Cheeger–Gromov. In particular, our result shows that the ρ–invariant is not a homotopy invariant for the manifolds in question.

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In their analysis of free involutions on the sphere, Browder and Livesay [4] introduced an invariant which they used to study the existence and uniqueness question for desuspensions. In particular, they produced infinitely many manifolds homotopy equivalent, but not diffeomorphic or even homeomorphic, to $\mathbb{RP}^{4k-1}$ for $k > 1$. Hirzebruch [11] soon gave an alternative definition of this invariant as follows: Any manifold M of dimension $4k-1$ with $\pi_1(M) = \mathbb{Z}_2$ has a multiple ℓM which bounds some W^{4k} such that $\pi_1(M) \to \pi_1(W)$ is an isomorphism. Define

$$\tau(M) = \frac{1}{\ell} \cdot g - \operatorname{sig}(\widetilde{W}) = \frac{1}{\ell}\left(\operatorname{tr} g\,|_{H_+} - \operatorname{tr} g\,|_{H_-}\right) = \frac{1}{\ell}\left(\operatorname{sig}(\widetilde{W}) - 2\operatorname{sig}(W)\right),$$

where $g \in \mathbb{Z}_2$ is the nontrivial element and $\widetilde{W}$ is the two-fold cover of W with $H_\pm \equiv H^{2k}_\pm(\widetilde{W})$. The fact that $\tau(M)$ is independent of W can be seen either using the G–signature formula [2] or by a bordism argument.

Remark Actually, Hirzebruch's invariant was only shown a bit later to be equal (up to a sign) to the Browder–Livesay invariant by Hirzebruch and Lopez de Medrano [13].

It is quite simple to modify τ to obtain invariants of arbitrary M^{4k-1} with finite fundamental group $G = \pi_1(M)$ by using the G–signature of $\widetilde{W}$ modulo the regular representation (as in [22]) or by using characters equal to $\operatorname{tr} g\,|_{H_+} - \operatorname{tr} g\,|_{H_-}$ for any $g \neq e$. As a very special and important case, one can even simply use $\operatorname{sig}(\widetilde{W}) - |G|\operatorname{sig}(W)$ which equals $\sum_{g\neq e}\chi_g(\operatorname{sig}_G(W))$ to obtain a quite useful invariant.

As an application of their index theorem for manifolds with boundary, Atiyah, Patodi and Singer [2] obtain such invariants of M without introducing W. More precisely, the APS invariant requires the use of an auxiliary Riemannian metric on M, but their index theorem for manifolds with boundary shows that the number obtained is independent of this metric. All of the numerical invariants mentioned above are differences of η–invariants (which measure "spectral asymmetry") for the signature operator twisted by suitable flat bundles.

The APS method admits extensive generalization to the case in which $\pi_1(M)$ is possibly infinite. In [1] Atiyah generalized the index theorem from compact closed manifolds to universal covers of compact manifolds (or even manifolds with proper actions of a discrete group). Using von Neumann traces, he defined

$$\operatorname{ind}_\Gamma(\widetilde{\not{D}}) = \dim_\Gamma(\ker\widetilde{\not{D}} \text{ on } L^2(\widetilde{M})) - \dim_\Gamma(\operatorname{coker}\widetilde{\not{D}} \text{ on } L^2(\widetilde{M})).$$

Both [14] and [15] are excellent references for ideas related to the von Neumann trace and its applications in geometry and topology.

Note Atiyah showed that $\mathrm{ind}_\Gamma(\widetilde{\not{D}}) = \mathrm{ind}(\not{D})$, which generalizes the *multiplicativity* property of ordinary indices for finite coverings (see appendix). This result is quite remarkable since, if the group is infinite, the elements of $\ker \not{D}$ never lift to be L^2 on the universal cover, but they nonetheless lead indirectly to elements of the L^2 kernel (at the level of indices, of course). Cheeger and Gromov [5, 6] introduced the analogue of the APS invariant in this setting: namely they studied an L^2–invariant $\eta_{(2)}(\widetilde{M})$ of compact M and the difference $\rho_{(2)}(M) = \eta_{(2)}(\widetilde{M}) - \eta(M)$ which is metric independent. For $\Gamma = \mathbb{Z}_2$ the quantity $\rho_{(2)}(M)$ is simply the Hirzebruch invariant or the Browder–Livesay invariant.

Thom's classical work on cobordism implies that every compact odd dimensional oriented manifold M has a multiple rM which is the boundary of an oriented manifold W. Hausmann [9] showed furthermore that, for every such M with fundamental group Γ, there is a manifold W such that Γ injects into $\pi_1(W)$ and $\partial W = rM$, for some multiple rM of M; that is, he showed that nullcobordant manifolds bound in such a way that their fundamental groups inject. If Γ is a group, we can define the L^2–signature $\mathrm{sig}^{\Gamma}_{(2)}(V) = \dim_\Gamma(V^+) - \dim_\Gamma(V^-)$ for any $\ell^2(\Gamma)$–module V endowed with nonsingular symmetric bilinear form. As usual, one uses an inner product to rewrite the symmetric bilinear form in terms of a self-adjoint operator, and then uses the spectral theorem to obtain projections to the positive and negative definite part. If N^{4k} is a Γ–space, let $\mathrm{sig}^{\Gamma}_{(2)}(N)$ be given by the L^2–signature of the symmetric form induced by cap product on its middle cohomology.

Define a new "Hirzebruch type" invariant $\tau_{(2)}$ given by

$$\tau^G_{(2)}(M) = \frac{1}{r}\left(\mathrm{sig}^{G}_{(2)}(W_G) - \mathrm{sig}\,(W)\right),$$

where $G = \pi_1(W)$ and W_G is the induced G–cover of W. Suppose that $rM = \partial W$ with $\pi_1(M) = \Gamma$ injecting into $\pi_1(W) = G$. If G injects into some larger group G', and let $W_{G'}$ be the G'–space induced from the G action on W_G to G'. Notice that $\partial(W_{G'}/G') = rM$, so we can use the larger group G' to define $\tau_{(2)}$. However, by the Γ–induction property of Cheeger–Gromov [6, page 8, equation (2.3)], we have

$$\begin{aligned}
\tau^{G'}_{(2)}(M) &= \tfrac{1}{r}\left(\mathrm{sig}^{G'}_{(2)}(W_{G'}) - \mathrm{sig}\,(W_{G'}/G')\right)\\
&= \tfrac{1}{r}\left(\mathrm{sig}^{G}_{(2)}(W) - \mathrm{sig}\,(W)\right)\\
&= \tau^{G}_{(2)}(M).
\end{aligned}$$

So one can pass to any larger group without changing the value of the invariant. Now, given two manifolds W and W' with the required bounding properties, we can use the large group $G' = \pi_1(W) *_\Gamma \pi_1(W')$, which contains both fundamental groups, and the usual Novikov additivity argument, to see that $\tau_{(2)}$ is independent of all choices.

In fact, according to [16], we have $\tau_{(2)} = \rho_{(2)}$, but we do not need this equality for the proofs of our main theorems. The goal of this paper is to prove a few theorems about $\tau_{(2)}(M)$ (or $\rho_{(2)}(M)$) that extend or are analogues of the classically understood situation for finite fundamental group.

Theorem 1 *For any compact oriented manifold M^{4k-1}, where $k \geq 2$, such that $\pi_1(M)$ is not torsion-free, there exist infinitely many manifolds M_i which are simple homotopy equivalent and tangentially equivalent to M, but not homeomorphic to M. If M is smooth, the M_i can be taken to be smooth as well. This infinite number is detected by $\tau_{(2)}$.*

Remarks

(1) Very similar arguments to the proof of Theorem 1 show that any M^{4k+2} with non-torsion-free fundamental group has infinite diffeotopy group.

(2) The analogue for M^{4k+1} is false. For instance, the real projective space $\mathbb{RP}^{4k+1}$ has a finite structure set. See [13]. Also, we cannot dispense with the orientability hypothesis as $\mathbb{RP}^2 \times S^5$ has finite structure set.

(3) There are predictions in the literature about $L_{4k}(\Gamma)\otimes\mathbb{Q}$ and its expression in terms of the group homology of normalizers of finite subgroups. See [8] and page 261 of [24]. These would suggest that Theorem 1 is true except for the thorny problem of self-homotopy equivalences of M. These maps $h : M \to M$ could represent nontrivial elements of the structure set $S(M)$, but nonetheless they are *not* the elements described in Theorem 1.

(4) If M is smooth, all the M_i can be taken smooth as well, without even being homeomorphic. The extension of $\tau_{(2)}$ from smooth to topological manifolds is not very hard by bordism methods. After multiplication by a positive integer, every topological manifold is cobordant to a smooth manifold N via a cobordism V (all with the same fundamental group), so we can consider $\tau_{(2)}(N) - (\mathrm{sig}_{(2)} - \mathrm{sig})(V)$.

(5) The theorem above asserts that, whenever $\pi_1(M)$ has torsion, then $\tau_{(2)}$ is not a homotopy invariant. Mathai [18] conjectured that the converse is true for $\rho_{(2)}$ and proved it for torsion-free crystallographic fundamental group. Thus, the above result is the converse to Mathai's conjecture.

Mathai's conjecture is an analogue of a conjecture and theorem of the second author [23] for Atiyah–Patodi–Singer invariants. Keswani [12] proved a general homotopy invariance theorem: if the assembly map $K_*(B\Gamma) \to K_*(C^*_{\max}\Gamma)$ is surjective, then $\rho_{(2)}$ is a homotopy invariant for manifolds with fundamental group Γ. This hypothesis holds for fundamental group of real and complex hyperbolic manifolds for torsion-free amenable groups by the theorem of Higson and Kasparov [10]. However, it fails for all nontrivial groups satisfying property T, ie, for many torsion-free lattices in higher rank or in $\mathrm{Sp}(n,1)$. Our next theorem somewhat repairs this problem.

Theorem 2 *If Γ is a torsion-free discrete subgroup of $SL_n(\mathbb{R})$, then ρ is a homotopy invariant for manifolds with fundamental group Γ. More generally, if $\Gamma = \pi_1(M)$ is residually finite and the Borel conjecture holds for Γ, then $\rho_{(2)}$ is a homotopy invariant (see eg [8]).*

In a future paper, the first author plans to remove the residual finiteness hypotheses arising in this work.

We now turn to the proof of Theorem 1. Let M be as in the theorem and $\mathbb{Z}_n \subset \pi_1(M) = \Gamma$ be a nontrivial cyclic subgroup. The M_i are just the results of acting by elements of $\mathrm{im}\,(L_{4k}(\mathbb{Z}_n) \to L_{4k}(\Gamma))$ on $S(M)$ in the surgery exact sequence

$$\cdots \to H_{4k}(M,\mathbb{L}) \to L_{4k}(\Gamma) \to S(M) \to H_{4k-1}(M,\mathbb{L}) \to L_{4k-1}(\Gamma) \to \cdots$$

The image of $A : H_{4k}(M,\mathbb{L}) \to L_{4k}(\Gamma)$ consists of quadratic forms arising from closed manifolds and for such manifolds we have $\mathrm{sig}^{\Gamma}_{(2)}(N) = \mathrm{sig}\,(N)$ by Atiyah [1]. Note that this higher signature may be viewed as a map $\mathrm{sig}^{\Gamma}_{(2)} : L_{4k}(\Gamma) \to \mathbb{R}$ (see [22] for the definition of L–groups). This image must equal $\ker\,(L_{4k}(\Gamma) \to S(M))$, the set of all $a \in L_{4k}(\Gamma)$ for which $a(M) = M$ diffeomorphically.[1] If $a \in L_{4k}(\Gamma)$, let V_a be the form representing it. Let $\alpha_\Gamma : L_{4k}(\Gamma) \to \mathbb{R}$ be given by the homomorphism $\alpha_\Gamma(V) \equiv \mathrm{sig}^{\Gamma}_{(2)}(V) - \mathrm{sig}\,(V)$. Hence, if $a(M) = M$, then $\alpha_\Gamma(V_a) = 0$. We would like to produce infinitely many $V_{a'}$ such that $\alpha_\Gamma(V_{a'}) \neq 0$.

To show that the homomorphism $\alpha_\Gamma : L_{4k}(\Gamma) \to \mathbb{R}$ has infinite image, it suffices to show that it is nontrivial. Let $p_* : L_{4k}(\mathbb{Z}_n) \to L_{4k}(\Gamma)$ be the map induced

[1] More precisely, the equality represents a diffeomorphism homotopic to the homotopy equivalence implicit in the fact that $a(M)$ is a member of a structure set.

by the injection $p : \mathbb{Z}_n \to \Gamma$. By the induction property of Γ–dimension, we have the commutative diagram

$$\begin{tikzcd}
L_{4k}(\mathbb{Z}_n) \arrow[r, "\mathrm{sig}^{\mathbb{Z}_n}_{(2)}"] \arrow[d, "p_*"'] & \mathbb{R} \\
L_{4k}(\Gamma) \arrow[ur, "\mathrm{sig}^{\Gamma}_{(2)}"'] &
\end{tikzcd}$$

so we only have to check that $\alpha_{\mathbb{Z}_n} : L_{4k}(\mathbb{Z}_n) \to \mathbb{R}$ is nontrivial. We note that, inverting 2, it makes no difference whether the coefficient ring is $\mathbb{Z}$ or $\mathbb{Q}$ [20] or whether one works with projective, free or based-free modules (by the well-known "Rothenberg exact sequences"). Thus, we need only produce a bilinear form on a projective module over $\mathbb{Q}\mathbb{Z}_n$, namely the one-by-one bilinear form [1] on the module $\mathbb{Q}$, which is projective over $\mathbb{Q}\mathbb{Z}_n$. By definition, our invariant is $\frac{1}{n} - 1$ for this element, which is always nontrivial for $n > 1$.

If rM cobounds some space W for which $\pi_1(M)$ injects into $G = \pi_1(W)$, we can extend each boundary component M by the cobordism Y, so that rM' cobounds W', where W' is obtained from W and r copies of Y. Hence by Novikov additivity and Γ–induction, we have

$$\begin{aligned}
\tau^G_{(2)}(M) - \tau^G_{(2)}(M') &= \tfrac{1}{r}\big(\mathrm{sig}^G_{(2)}(W_G) - \mathrm{sig}\,(W)\big) - \tfrac{1}{r}\big(\mathrm{sig}^G_{(2)}(W'_G) - \mathrm{sig}\,(W')\big) \\
&= \tfrac{1}{r}\big(\mathrm{sig}^G_{(2)}(W_G) - \mathrm{sig}\,(W)\big) - \tfrac{1}{r}\big(\mathrm{sig}^G_{(2)}(W)) \\
&\qquad\qquad + r \cdot \mathrm{sig}^G_{(2)}(Y_G - \mathrm{sig}\,(W) - r \cdot \mathrm{sig}\,(Y)\big) \\
&= \mathrm{sig}^G_{(2)}(Y_G) - \mathrm{sig}\,(Y) \\
&= \mathrm{sig}^\Gamma_{(2)}(Y_\Gamma) - \mathrm{sig}\,(Y) \\
&= \alpha_\Gamma(V),
\end{aligned}$$

where $V \in L_{4k}(\Gamma)$ represents the cobordism Y.

Let $V_a \in L_{4k}(\Gamma)$ such that $\alpha_\Gamma(V_a) \neq 0$, where $\Gamma = \pi_1(M)$. Since $(M \to M) \in S_{4k-1}(M)$, there is a cobordism Y_a such that $\partial Y_a = M \cup M'$ representing V_a. Clearly M and M' are homotopy equivalent but not diffeomorphic. Consider $V_{a_i} \in L_{4k}(\Gamma)$ for all $i \in \mathbb{N}$ such that $\alpha_\Gamma(V_{a_i}) \neq \alpha_\Gamma(V_{a_j})$ for all $i \neq j$ and all nonzero. Let M_i be the result of acting on M by V_{a_i}. Then $\tau^G_{(2)}(M) - \tau^G_{(2)}(M_i) = \alpha_\Gamma(V_{a_i}) \neq 0$. Also

$$\begin{aligned}
\tau^G_{(2)}(M_i) - \tau^G_{(2)}(M_j) &= \tau^G_{(2)}(M_i) - \tau^G_{(2)}(M) + \tau^G_{(2)}(M) - \tau^G_{(2)}(M_j) \\
&= -\alpha_\Gamma(V_{\alpha_i}) + \alpha_\Gamma(V_{\alpha_j}) \neq 0,
\end{aligned}$$

as desired. □

Now we prove Theorem 2. For a finite group Γ, the rho-invariant $\rho_{(2)} = \eta_{(2)} - \eta$ is equal to $\frac{1}{|\Gamma|}\eta_{\otimes\mathbb{R}\Gamma} - \eta$, where $\eta_{\otimes\mathbb{R}\Gamma}$ is the eta–invariant for M with coefficients in the flat bundle associated to the regular representation. According to [17], if Γ is residually finite with descending quotients Γ_k, then

$$\rho_{(2)} = \lim_{k\to\infty}\left(\frac{1}{|\Gamma_k|}\left(\eta_{\otimes\mathbb{R}\Gamma} - \eta\right)\right).$$

Indeed, Cheeger and Gromov introduced $\rho_{(2)}$ to generalize the right-hand side to non-residually finite fundamental group. However, according to [24], for *any* representation V of a group satisfying the Borel conjecture, the quantity $\frac{1}{\dim V}\eta_{\otimes V} - \eta$ is a homotopy invariant. Since ρ is the limit of homotopy invariants, a fortiori, it too is a homotopy invariant. □

If Γ is a discrete subgroup of $\mathrm{SL}_n(\mathbb{R})$, then it is a classical lemma of Selberg that Γ is residually finite. The fact that the Borel conjecture holds if Γ is in addition torsion-free is the main result of [8].

Problem If $\pi_1(M)$ has many conjugacy classes of elements of finite order, then one would expect that $S(M)$ is rather larger than the $\mathbb{Z}$ we just detected. Can this statement be proven unconditionally? Wolfgang Lück has pointed out that, using the traces associated to elements of finite order that have finitely many conjugates (eg, central elements), one can extend Theorem 1 and detect a larger $S(M)$. But this result is still rather smaller than the predicted size of the structure set.

Appendix: Acyclic groups and Atiyah's theorem

For the convenience of the reader we shall prove here the theorems of Atiyah and Hausmann invoked in the paper. Let M be a closed manifold. According to [3], there is an acyclic group A with an injective homomorphism $h\colon \pi_1(M) \to A$. Since A is acyclic, the map $\Omega_n(K(A,1)) \to \Omega_n(*)$ is an isomorphism, so that, if M bounds (ie, M is zero in $\Omega_n(*)$), it bounds a manifold W such that

$$\begin{tikzcd} M \arrow[r] \arrow[d] & K(\pi_1(M),1) \arrow[d] \\ W \arrow[r] & K(A,1) \end{tikzcd}$$

commutes; *a fortiori* $\pi_1(M) \to \pi_1(W)$ is injective. This argument gives Hausmann's result.

Now for Atiyah's theorem we observe, by the induction and bordism invariance properties of $\mathrm{sig}_{(2)}$, that we have the commutative diagram

$$\begin{tikzcd}
& \Omega_{4k}(*) \arrow[dl, "\mathrm{sig}"'] \arrow[d] \arrow[dr, "\mathrm{sig}_{(2)}"] & \\
\mathbb{Z} & \Omega_{4k}\big(K(\pi,1)\big) \arrow[l, "\mathrm{sig}"] \arrow[r, "\mathrm{sig}^{\pi}_{(2)}"'] \arrow[d] & \mathbb{R} \\
& \Omega_{4k}\big(K(A,1)\big) \arrow[ul, "\mathrm{sig}"] \arrow[ur, "\mathrm{sig}^{A}_{(2)}"'] &
\end{tikzcd}$$

Since $\Omega_{4k}(*) \to \Omega_{4k}(K(A,1))$ is an isomorphism and $\mathrm{sig} = \mathrm{sig}_{(2)}$ for the trivial group, the conclusion follows.

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