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Strongly fillable contact 3–manifolds without Stein fillings

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Abstract

We use the Ozsváth–Szabó contact invariant to produce examples of strongly symplectically fillable contact 3–manifolds which are not Stein fillable.

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1 Introduction

There is a strong relationship between contact topology and symplectic topology due to the fact that contact structures provide natural boundary conditions for symplectic structures on manifolds with boundary. Given a contact manifold (Y,ξ) and a symplectic manifold (W,ω) with $\partial W = Y$, we say that (W,ω) *fills* (Y,ξ) if some compatibility conditions are satisfied. Depending on how restricting these conditions are, there are several different notions of fillability. The most widely studied in the literature are weak or strong symplectic fillability and Stein fillability.

In the following we will always assume Y is an oriented 3–manifolds and ξ is oriented and positive. This means that ξ is the kernel of a globally defined smooth 1–form α on Y such that $\alpha \wedge d\alpha$ is a volume form inducing the fixed orientation of Y.

Definition 1.1 A contact manifold (Y,ξ) is *weakly symplectically fillable* if Y is the boundary of a symplectic manifold (W,ω) with $\omega|_\xi > 0$.

Since ω orients W and ξ orients Y, we also require that the orientation of Y as boundary of W coincides with the orientation induced by ξ.

Definition 1.2 A contact manifold (Y,ξ) is *strongly symplectically fillable* if Y is the boundary of a symplectic manifold (W,ω) and ξ is the kernel of a smooth 1–form α on Y such that $\omega|_Y = d\alpha$.

Definition 1.3 A *Stein manifold* is a complex manifold (X,J) with a proper function $\varphi\colon X \to [0,+\infty)$ such that $dJ^*(d\varphi)$ is a Kähler form on X.

Definition 1.4 A contact manifold (Y,ξ) is *Stein fillable* (or *holomorphically fillable*) if Y is the boundary of a domain $W = \varphi^{-1}([0,t])$ in a Stein manifold (X,J) for some regular value t of φ, and ξ is the field of the complex hyperplanes of $J|_{\partial W}$.

Remark In the literature there are several different equivalent definitions of Stein manifold: see for example [4, Section 4].

There are obvious inclusions

$$\begin{Bmatrix}\text{Stein}\\ \text{Fillable}\end{Bmatrix} \subset \begin{Bmatrix}\text{Strongly}\\ \text{Fillable}\end{Bmatrix} \subset \begin{Bmatrix}\text{Weakly}\\ \text{Fillable}\end{Bmatrix}$$

moreover, weakly fillable contact structures are tight by a deep theorem of Eliashberg and Gromov [2, 10]. The goal of this article is to prove that the inclusion

$$\begin{Bmatrix} \text{Stein} \\ \text{Fillable} \end{Bmatrix} \subset \begin{Bmatrix} \text{Strongly} \\ \text{Fillable} \end{Bmatrix}$$

is strict in dimension three. Let $-\Sigma(2,3,6n+5)$ be the 3–manifold defined by the surgery diagram in Figure 1. We will prove the following theorem.

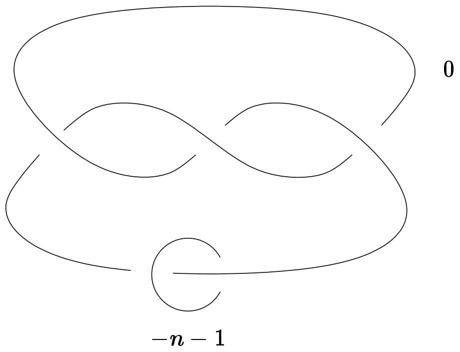

Figure 1: The surgery diagram of $-\Sigma(2,3,6n+5)$

Theorem 1.5 *For any $n \geq 2$ and even, the 3–manifold $-\Sigma(2,3,6n+5)$ admits a strongly symplectically fillable contact structure which is not Stein fillable.*

All other inclusions have already been proved to be strict: tight but non weakly fillable contact structures have been found first by Etnyre and Honda [5] and later by Lisca and Stipsicz [13, 14]. A weakly fillable but non Strongly fillable contact structure has been found first by Eliashberg [3] and later more have been found by Ding and Geiges [1].

The main tool used in this article is the contact invariant in Heegaard–Floer theory recently introduced by Ozsváth and Szabó [15].

2 Construction of the non Stein fillable contact manifolds

Let M_0 be the 3–manifold obtained by 0–surgery on the right-handed trefoil knot. M_0 admits a presentation as a T^2–bundle over S^1 with monodromy map

$$A\colon\ T^2 \times \{1\} \to T^2 \times \{0\}$$

given by $A = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$. Put coordinates (x, y, t) on $T^2 \times \mathbb{R}$. The 1–forms

$$\alpha_n = \sin(\phi(t))dx + \cos(\phi(t))dy$$

on $T^2 \times \mathbb{R}$ define contact structures ξ_n on M_0 for any $n > 0$ provided that

(1) $\phi'(t) > 0$ for any $t \in \mathbb{R}$

(2) α_n is invariant under the action $(\mathbf{v}, t) \mapsto (A\mathbf{v}, t-1)$

(3) $n\pi \leq \sup_{t\in\mathbb{R}}(\phi(t+1) - \phi(t)) < (n+1)\pi$.

The main result about this family of contact structures we will need in the present article is the following.

Theorem 2.1 ([8, Proposition 2 and Theorem 6], [1, Theorem 1]) *The contact structures ξ_n do not depend on the function ϕ up to isotopy, and are all weakly symplectically fillable.*

Let F be the image in M_0 of the segment $\{0\} \times [0,1] \subset T^2 \times [0,1]$, then F is Legendrian with respect to the contact structure ξ_n for all n. Denote by K the right-handed trefoil knot in S^3. We can choose a diffeomorphism from the complement of a tubular neighbourhood of K in S^3 to the complement of a tubular neighbourhood of F in M_0 so that the meridian of K is mapped to a longitude of F. This diffeomorphism defines a framing on F, and the framing so defined allows us to define a twisting number for F.

Lemma 2.2 [7, Lemma 3.5] *The twisting number of ξ_n along the Legendrian curve F is $tn(F, \xi_n) = -n$*

Legendrian surgery on (M_0, ξ_n) along F is smoothly equivalent to the surgery described by Figure 1 which produces the manifold $-\Sigma(2,3,6n+5)$. We denote the tight contact structure on $-\Sigma(2,3,6n+5)$ obtained by Legendrian surgery on (M_0, ξ_n) along F by η_0. The following theorem proves the strong fillability part of Theorem 1.5.

Theorem 2.3 *The contact manifolds $(-\Sigma(2,3,6n+5), \eta_0)$ are strongly symplectically fillable for any $n \geq 1$.*

Proof The contact manifolds (M_0, ξ_n) are weakly symplectically fillable by Theorem 2.1. Since Legendrian surgery preserves weak fillability by [6, Theorem 2.3], $(-\Sigma(2,3,6n+5), \eta_0)$ is also weakly fillable. Since the manifolds $\Sigma(2,3,6n+5)$ are homology spheres, by [4, Proposition 4.1] the symplectic form on the filling can be modified in a neighbourhood of the boundary so that the filling becomes strong. □

The non Stein fillability part of Theorem 1.5 can now be made more precise with the following statement.

Theorem 2.4 *The contact manifolds $(-\Sigma(2,3,6n+5),\eta_0)$ are not Stein fillable for any $n \geq 2$ and even.*

The proof of this theorem is the goal of Section 4.

3 Overview of the contact invariant

In this section we give a brief overview of Heegaard–Floer homology and of the contact invariant defined by Ozsváth and Szabó. We will not treat the subject in its most general form, but only in the form it will be needed in the proof of Theorem 2.4.

3.1 Heegaard–Floer homology

Heegaard–Floer homology is a family of topological quantum field theories for *Spin*^{c} 3–manifolds introduced by Ozsváth and Szabó in [16, 18, 19]. In their simpler form they associate vector spaces $\widehat{HF}(Y,\mathfrak{t})$ and $HF^+(Y,\mathfrak{t})$ over $\mathbb{Z}/2\mathbb{Z}$ to any closed oriented *Spin*^{c} 3–manifold $(Y,\mathfrak{t})$, and homomorphisms

$$F^\circ_{W,\mathfrak{s}}\colon HF^\circ(Y_1,\mathfrak{t}_1) \to HF^\circ(Y_2,\mathfrak{t}_2)$$

to any oriented *Spin*^{c}–cobordism $(W,\mathfrak{s})$ between two *Spin*^{c}–manifolds $(Y_1,\mathfrak{t}_1)$ and $(Y_2,\mathfrak{t}_2)$ such that $\mathfrak{s}|_{Y_i} = \mathfrak{t}_i$. Here HF° denotes either $\widehat{HF}$ or HF^+. The groups $\widehat{HF}(Y,\mathfrak{t})$ and $HF^+(Y,\mathfrak{t})$ are linked to one another by the exact triangle

$$\begin{tikzcd} \widehat{HF}(Y,\mathfrak{t}) \arrow[r] & HF^+(Y,\mathfrak{t}) \arrow[r] & HF^+(Y,\mathfrak{t}) \arrow[ll, bend left] \end{tikzcd} \tag{1}$$

This exact triangle is natural in the sense that its maps commute with the maps induced by cobordisms.

It was shown in [16] that, when $c_1(\mathfrak{t})$ is a torsion element, the vector spaces $\widehat{HF}(Y,\mathfrak{t})$ and $HF^+(Y,\mathfrak{t})$ come with a $\mathbb{Q}$–grading. In conclusion, for a torsion *Spin*^{c}–structure $\mathfrak{t}$ on Y the Heegaard–Floer homology groups $HF^\circ(Y,\mathfrak{t})$ split as

$$HF^\circ(Y,\mathfrak{t}) = \bigoplus_{d\in\mathbb{Q}} HF^\circ_{(d)}(Y,\mathfrak{t}).$$

The set of the $Spin^c$–structures on a manifold has an involution called *conjugation.* Given a $Spin^c$–structure $\mathfrak{t}$, we denote its conjugate $Spin^c$–structure by $\bar{\mathfrak{t}}$. We have $c_1(\bar{\mathfrak{t}}) = -c_1(\mathfrak{t})$. There is an isomorphism $\mathfrak{J}\colon HF^\circ(Y,\mathfrak{t}) \to HF^\circ(Y,\bar{\mathfrak{t}})$ defined in [18, Theorem 2.4]. We recall that the isomorphism $\mathfrak{J}$ preserves the $\mathbb{Q}$–grading of the Heegaard–Floer homology groups when $c_1(\mathfrak{t})$ is a torsion cohomology class, and is a natural transformation in the following sense.

Proposition 3.1 [16, Theorem 3.6] *Let $(W,\mathfrak{s})$ be a $Spin^c$–cobordism between $(Y_1,\mathfrak{t}_1)$ and $(Y_2,\mathfrak{t}_2)$. Then the following diagram*

$$\begin{array}{ccc} HF^\circ(Y_1,\mathfrak{t}_1) & \xrightarrow{F^\circ_{W,\mathfrak{s}}} & HF^\circ(Y_2,\mathfrak{t}_2) \\ \Big\downarrow \mathfrak{J} & & \Big\downarrow \mathfrak{J} \\ HF^\circ(Y_1,\bar{\mathfrak{t}}_1) & \xrightarrow{F^\circ_{W,\bar{\mathfrak{s}}}} & HF^\circ(Y_2,\bar{\mathfrak{t}}_2) \end{array}$$

commutes.

The isomorphism $\mathfrak{J}$ commutes also with the maps in the exact triangle (1).

3.2 Contact invariant

A contact structure ξ on a 3–manifold Y determines a $Spin^c$–structure $\mathfrak{t}_\xi$ on Y such that $c_1(\mathfrak{t}_\xi) = c_1(\xi)$. To any contact manifold (Y,ξ) we can associate an element $c(\xi) \in \widehat{HF}(-Y,\mathfrak{t}_\xi)$ which is an isotopy invariant of ξ, see [15]. Sometimes it is also useful to consider the image $c^+(\xi) \in HF^+(-Y,\mathfrak{t}_\xi)$ of $c(\xi)$ under the map $\widehat{HF}(-Y,\mathfrak{t}_\xi) \to HF^+(-Y,\mathfrak{t}_\xi)$ in the exact triangle (1). The Ozsváth–Szabó contact invariant satisfies the following properties.

Theorem 3.2 [15, Theorem 1.4 and Theorem 1.5] *If (Y,ξ) is overtwisted, then $c(\xi)=0$. If (Y,ξ) is Stein fillable, then $c(\xi)\neq 0$.*

Proposition 3.3 [15, Proposition 4.6] *If $c_1(\xi)$ is a torsion homology class, then $c(\xi)$ is a homogeneous element of degree $-d_3(\xi)-\frac{1}{2}$, where $d_3(\xi)$ denotes the 3–dimensional homotopy invariant introduced by Gompf [9, Definition 4.2].*

Theorem 3.4 [20, Theorem 4] *Let W be a smooth compact 4–manifold with boundary $Y=\partial W$. Let J_1, J_2 be two Stein structures on W that induce $Spin^c$–structures $\mathfrak{s}_1$, $\mathfrak{s}_2$ on W and contact structures ξ_1, ξ_2 on Y. We puncture W and regard it as a cobordism from $-Y$ to S^3. Suppose that $\mathfrak{s}_1|_Y$ is isotopic to $\mathfrak{s}_2|_Y$, but the $Spin^c$–structures $\mathfrak{s}_1$, $\mathfrak{s}_2$ are not isomorphic. Then*

(1) $F^+_{W,\mathfrak{s}_i}(c(\xi_j)) = 0$ *for* $i \neq j$;

(2) $F^+_{W,\mathfrak{s}_i}(c(\xi_i))$ *is a generator of* $HF^+(S^3)$.

The space of oriented contact structures on Y has a natural involution called *conjugation*. For any contact structure ξ on a 3–manifold Y we denote by $\overline{\xi}$ the contact structure on Y obtained from ξ by inverting the orientation of the planes. The conjugation of contact structures is compatible with the conjugation of the $Spin^c$–structure defined by the contact structure, in fact $\mathfrak{t}_{\overline{\xi}} = \overline{\mathfrak{t}_\xi}$. The contact invariant behaves well with respect to conjugation.

Proposition 3.5 [7, Theorem 2.10] *Let* (Y, ξ) *be a contact manifold, then*

$$c(\overline{\xi}) = \mathfrak{J}(c(\xi)).$$

4 Proof of the non fillability of $(-\Sigma(2,3,6n+5), \eta_0)$

In this article we will consider only integer homology spheres, which have therefore a unique $Spin^c$–structure. For this reason from now on we will always suppress the $Spin^c$–structure in the notation of the Heegaard–Floer groups.

The key ingredients in the proof of Theorem 2.4 are the conjugation invariance of η_0 and the structure of the $\mathfrak{J}$–action on $\widehat{HF}(\Sigma(2,3,6n+5))$. The starting point is a general observation about the Stein fillings of conjugation invariant contact structures.

Proposition 4.1 *Let* ξ *be a contact structure on a* 3*–manifold* Y *which is isotopic to its conjugate* $\overline{\xi}$. *If* (W, J) *is a Stein filling of* ξ *and* $\mathfrak{s}$ *is its canonical* $Spin^c$*–structure, then* $\mathfrak{s}$ *is isomorphic to its conjugate* $\overline{\mathfrak{s}}$.

Proof If (W, J) is a Stein filling of ξ, then $(W, -J)$ is a Stein filling of $\overline{\xi}$, and the canonical $Spin^c$–structure of $(W, -J)$ is $\overline{\mathfrak{s}}$. Puncture W and regard it as a cobordism between $-Y$ and S^3. Since $\overline{\xi}$ is isotopic to ξ we have

$$F_{W,\mathfrak{s}}(c(\xi)) = F_{W,\overline{\mathfrak{s}}}(c(\xi)) \neq 0.$$

Theorem 3.4 implies that $\mathfrak{s}$ is isomorphic to $\overline{\mathfrak{s}}$. □

Remark Proposition 4.1 can be deduced also from Seiberg–Witen theory, see for example [12, Theorem 1.2] or [11, Theorem 1.2].

By [7, Theorem 3.12] the 3–dimensional homotopy invariant of η_0 is $d_3(\eta_0) = -\frac{3}{2}$, therefore the contact invariant $c(\eta_0)$ belongs to $\widehat{HF}_{(+1)}(\Sigma(2,3,6n+5))$. The group $HF^+(-\Sigma(2,3,6n+5))$ is computed in [17, Section 8]. From this it is easy to prove that $\widehat{HF}_{(+1)}(\Sigma(2,3,6n+5))$ is isomorphic to $(\mathbb{Z}/2\mathbb{Z})^n$ by applying the exact triangle (1) and the isomorphism $\widehat{HF}_{(d)}(Y) \cong \widehat{HF}_{(-d)}(-Y)$ which holds for any homology sphere Y.

Now we give a closer look at the action of $\mathfrak{J}$ on $\widehat{HF}_{(+1)}(\Sigma(2,3,6n+5))$ by considering the action of conjugation on a set of Stein fillable contact structures on $-\Sigma(2,3,6n+5)$. For any $n \in \mathbb{N}$ and $n \geq 2$ we define

$$\mathcal{P}_n^* = \{-n+1, -n+3, \ldots, n-3, n-1\}.$$

If n is even, then $0 \notin \mathcal{P}_n^*$. Given $i \in \mathcal{P}_n^*$, by η_i we denote the contact structure on $-\Sigma(2,3,6n+5)$ obtained by Legendrian surgery on the Legendrian link in the standard S^3 shown in Figure 2. In the following we will always assume n even, so there is no confusion between η_0 as defined in Section 2 and η_i with $i \in \mathcal{P}_n^*$. The contact structures η_i with $i \in \mathcal{P}_n^*$ are all Stein fillable and pairwise homotopic with 3–dimensional homotopy invariant $d_3(\eta_i) = -\frac{3}{2}$.

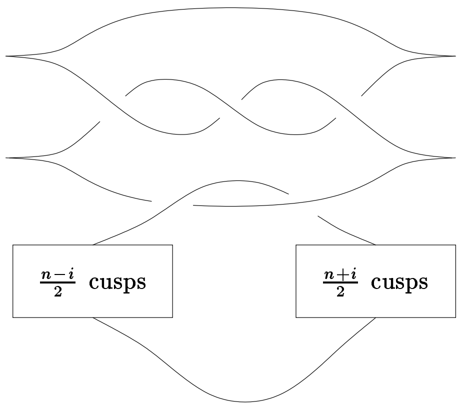

Figure 2: Legendrian surgery presentation of the contact manifold $(-\Sigma(2,3,6n+5), \eta_i)$ for $i \in \mathcal{P}_n^*$

Proposition 4.2 [20, Section 4] *The contact invariants $c(\eta_i)$ for $i \in \mathcal{P}_n^*$ generate $\widehat{HF}_{(+1)}(\Sigma(2,3,6n+5))$.*

Proposition 4.3 [7, Proposition 3.8] *The contact structure $\overline{\eta_i}$ obtained from η_i by conjugation is isotopic to η_{-i} for $i \in \mathcal{P}_n^*$, and η_0 is isotopic to its conjugate $\overline{\eta_0}$.*

Putting Proposition 4.2 and Proposition 4.3 together we obtain the following lemma.

Lemma 4.4 *If n is even, then the subspace*

$$\mathit{Fix}(\mathfrak{J}) \subset \widehat{HF}_{(+1)}(\Sigma(2,3,6n+5))$$

of the fix points for the action of $\mathfrak{J}$ on $\widehat{HF}_{(+1)}(\Sigma(2,3,6n+5))$ is generated by $c(\eta_i)+c(\eta_{-i})$ for $i \in \mathcal{P}_n^*$.

Proof Let $x \in \mathrm{Fix}(\mathfrak{J})$ be a fixed point. We write

$$x = \sum_{i \in \mathcal{P}_n^*} \alpha_i c(\eta_i)$$

for $\alpha_i \in \{0,1\}$, then applying $\mathfrak{J}$ we obtain

$$x = \sum_{i \in \mathcal{P}_n^*} \alpha_i c(\eta_{-i}).$$

From this we deduce that $\alpha_i = \alpha_{-i}$, which implies the lemma. □

Proof of Theorem 2.4 Suppose (W,J) is a Stein filling of $(-\Sigma(2,3,6n+5),\eta_0)$ and call $\mathfrak{s}$ its canonical Spin^c-structure. By Proposition 4.1 $\mathfrak{s}$ is invariant under conjugation. Moreover, $c(\eta_0) \in \mathrm{Fix}(\mathfrak{J})$ by Proposition 3.5, therefore $c(\eta_0)$ is a linear combination of elements of the form $c(\eta_i)+c(\eta_{-i})$ for $i \in \mathcal{P}_n^*$. Applying the map $\widehat{HF}(\Sigma(2,3,6n+5)) \to HF^+(\Sigma(2,3,6n+5))$ we obtain that $c^+(\eta_0)$ is a linear combination of elements of the form $c^+(\eta_i)+c^+(\eta_{-i})$.

Puncture the Stein filling W and regard it as a cobordism from $\Sigma(2,3,6n+5)$ to S^3. Applying $F^+_{W,\mathfrak{s}}$ to each $c^+(\eta_i)+c^+(\eta_{-i})$ we get

$$F^+_{W,\mathfrak{s}}(c^+(\eta_i)+c^+(\eta_{-i})) = F^+_{W,\mathfrak{s}}(c^+(\eta_i)) + F^+_{W,\mathfrak{s}}(\mathfrak{J}(c^+(\eta_i)) = 2F^+_{W,\mathfrak{s}}(c^+(\eta_i)) = 0$$

because

$$F^+_{W,\mathfrak{s}}(\mathfrak{J}(c^+(\eta_i)) = \mathfrak{J}(F^+_{W,\bar{\mathfrak{s}}}(c^+(\eta_i))) = F^+_{W,\mathfrak{s}}(c^+(\eta_i))$$

by Proposition 4.1, the naturality of the homomorphism $\mathfrak{J}$, and the triviality of the $\mathfrak{J}$–action on $HF^+(S^3)$. This implies $F^+_{W,\mathfrak{s}}(\eta_0)=0$, which is a contradiction with Theorem 3.4(2), therefore a Stein filling of $(-\Sigma(2,3,6n+5),\eta_0)$ cannot exist. □

Remark With the same argument we can actually prove that $(-\Sigma(2,3,6n+5),\eta_0)$ has no symplectic filling with exact symplectic form when n is even. We will call such a filling an *exact filling.* Exact fillability is a notion of fillability which is intermediate between strong and Stein fillability and has not been studied much yet. We do not know at present if exact fillability is a different notion from Stein fillability.

This stronger form of Theorem 2.4 can be proved by extending Theorem 3.4 to exact fillings.

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