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# Some computational results on mod 2 finite-type invariants of knots and string links

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**Abstract** We publish a table of primitive finite-type invariants of order less than or equal to six, for knots of ten or fewer crossings. We note certain mod-2 congruences, one of which leads to a chirality criterion in the Alexander polynomial. We state a computational result on mod-2 finite-type invariants of 2-strand string links.

**AMS Classification** 57M27, 57M25

**Keywords** Vassiliev invariants, finite-type invariants, chirality, Alexander polynomial, string links, 2-torsion

## 1 Introduction

In [11], Vassiliev described a new way to obtain invariants of knots in $\mathbb{S}^3$. His paper contains the outline of an algorithm for computing his invariants. Gusarov [5] obtained the same set of invariants independently and by different methods. These invariants of Vassiliev and Gusarov are now often referred to as finite-type invariants.

At the end of this paper there are tables of primitive invariants of order $\leq 6$ for knots of $\leq 10$ crossings. These invariants were computed using an implementation of Vassiliev's algorithm, which is described in [9] . In order to create tables such as these, a basis must be chosen for the invariants of order $\leq 6$. In Section 2 we will discuss the choice of such a basis, and make some related observations. In Section 3 we note that the algorithm for computing knot invariants extends easily to the computation of finite-type invariants of string links. We describe a computation which shows that there is a mod-2 weight system of order 5 (first noted by Kneissler and Dogolazky) for 2-strand string links which does not "integrate" to a mod-2 finite-type invariant of order 5. In Section 4 we present the two matrices for translating our numbers into finite-type invariants obtained from the derivatives of knot polynomials, following the notation of Kanenobu [7].

## 2 Choosing a basis

For a general reference on finite-type invariants, see Bar-Natan [1] or Birman [3].

Any $\mathbb{Q}$-valued knot invariant $v$ may be extended to singular knots in a unique way by the usual formula:

$$v(K_\times) = v(K_+) - v(K_-) \tag{1}$$

If there exists a positive integer $n$ such that $v(K) = 0$ for any knot $K$ with more than $n$ double points, then $v$ is said to be a finite-type invariant. The least such $n$ is called the order or type or degree of $v$. Let $V_n^*$ be the $\mathbb{Q}$ vector space of knot invariants of order $\leq n$. $V_n^*$ is finite dimensional because there are a finite number of chord diagrams with $n$ chords or less. Let $W_n^* \subset V_n^*$ be the subspace of invariants which are additive under the connected sum of knots. (That is, $w(K_1 \# K_2) = w(K_1) + w(K_2)$ for all $w \in W_n^*$ and for all knots $K_1, K_2$.) Additive finite-type invariants are often referred to as primitive invariants because they are the primitive elements of the graded Hopf algebra $\cup_{i=0}^{\infty} V_n^*$. This means that all finite-type invariants are linear combinations of products of the primitive ones. Therefore, in making tables of invariants, it suffices to list only basis elements for $W_n^*$.

There does not seem to be a canonical way to choose a basis for $W_n^*$. We list some of the desirable properties that such a basis $B_n = \{b_1, b_2, \ldots b_{\dim(W_n^*)}\}$ might have:

(1) $B_n$ should consist of $\mathbb{Z}$-valued invariants.

(2) $B_n$ should be a basis over $\mathbb{Z}$ for the primitive integer-valued invariants of order $\leq n$.

(3) For each $i \leq n$, the set $\{b_1, b_2, \ldots b_{\dim(W_i^*)}\}$ should be a basis for $W_i^*$.

(4) Knots of small crossing number should have small values on the basis invariants. I first did the computations presented here in 1992, and at that time an essentially random basis was obtained from the computer program that solved the T4T relations. A table of invariants using that basis was made available electronically, though it was never published. That table contained many four-digit numbers. Recently I have been able by ad-hoc methods to change the basis to give the values shown below, where the largest absolute value occuring is 39.

(5) If $w$ is an even-order basis element, then $w(m(K)) = w(K)$ for any knot $K$, where $m(K)$ denotes the mirror image of $K$. If $w$ is an odd-order basis element, then $w(m(K)) = -w(K)$ for any knot $K$. This is always

possible over $\mathbb{Q}$ (in fact over any ring where 2 is invertible, as noted by Vassiliev [11]), using the identity $w(K) = \frac{1}{2}(w(K) + w(m(K))) + \frac{1}{2}(w(K) - w(m(K)))$. The first term on the right will always have even order, and the second will always have odd order, so $v$ may be replaced by one of the two terms (modulo invariants of lower order), depending on whether the order of $v$ itelf is even or odd.

Even though each of the above conditions can be satisfied individually, it is not possible to satisfy them all at once. The basis invariants below satisfy all the conditions except 2 and they *almost* satisfy 2. The vectors in $\mathbb{Z}^{12}$ that actually occur as the values for specific knots form a sublattice of $\mathbb{Z}^{12}$ of index 16, reflecting four inevitable mod-2 congruences imposed by Condition 5. The basis is chosen so that that $v_3 \equiv v_{4a}$ modulo 2, and likewise $v_{5a} \equiv v_{6a}$, $v_{5b} \equiv v_{6b}$, and $v_{5c} \equiv v_{6c}$.

We note that $v_{4a} = \frac{1}{2}(3a_2 - a_2^2) + a_4$, where $\sum a_i x^i$ (with $i$ even) denotes the Conway polynomial of a knot. If $v_3(K)$ is odd, then $v_3(K) \neq 0$, and therefore $K$ is chiral. Hence if $\frac{1}{2}(3a_2 - a_2^2) + a_4$ is odd (or, to make it slightly more simple, if $\frac{1}{2}(a_2 + a_2^2) + a_4$ is odd), then $K$ is chiral. It is interesting to find this chirality criterion contained in the Conway polynomial. Stoimenow [10] has recently studied the chirality information in the determinant of a knot, which is the integer obtained by evaluating the Conway polynomial at $x = 2\sqrt{-1}$. The chirality criterion from $v_{4a}$ modulo 2 is independent of determinant. More precisely, given a number $d$ which occurs as the determinant of some knot, there exist knots $K, K'$, each with determinant $d$, such that $v_{4a}(K) \equiv 0$ and $v_{4a}(K') \equiv 1$, modulo 2. To see this, note that replacing $a_4$ by $a_4 + 1$ and $a_2$ by $a_2 + 4$ does not change the determinant of a knot, but it does change $v_{4a}$ modulo 2. Such a change can be made because any even polynomial with constant term equal to 1 occurs as the Conway polynomial of some knot.

## 3 Mod-2 invariants of 2-strand string links

Let $\mathbb{D}^2$ be the two-dimensional disk, and let $p_1, p_2 \dots p_k \in \mathbb{D}^2$ be $k$ distinct points. A $k$-strand string link is a $k$-tuple of disjoint tame curves in $\mathbb{D}^2 \times [0,1]$ such that the endpoints of the $i$th curve are $(p_i, 0)$ and $(p_i, 1)$ for all $1 \leq i \leq k$. The components of a string links are thus ordered, and each component has an unambiguous orientation. A string link may be given by a planar diagram, just as in the case of knots, and the three usual Reidemeister moves suffice to generate equivalence. We may also consider the larger set of singular string links, which are allowed to contain a finite number of double point singularities, just as in the case of knots. (Two extra Reidemeister moves are needed here, see [9].) Applying Relation 1, we may define a string link invariant $v$ to be of finite-type if there exists a positive integer $n$ such that $v$ vanishes on string links with more than $n$ singularities. The least such integer $n$ is called the order or type or degree of $v$.

Rather than deal with finite-type invariants directly, it is often convenient to consider the abelian group generated by all singular string links, subject to Relation 1 and to the relation that $v(L) = 0$ if $L$ has more than $n$ singularities. (Both of these are of course infinite families of relations.) For $k$-component string links, we denote this group by $V_n(k)$. The set of finite-type invariants of order $\leq n$ taking values in an abelian group $G$ is then identified with $\mathrm{Hom}(V_n(k), G)$ in the obvious way.

Although $V_n(k)$ is defined by an infinite presentation, it is well-known, and easily seen, that $V_n(k)$ is a finitely-generated abelian group. The following is a version of the well-known "fundamental theorem of Vassiliev invariants" (see Bar-Natan and Stoimenow [2]), and is easy to prove using a the same methods as in [9]:

**Theorem** *A presentation of $V_n(k)$ is given by any set $S$ which contains exactly one string link for each chord diagram with $\leq n$ chords, subject to the topological 4-term and 1-term relations, exactly one such relation from each configuration class of order $\leq n$.*

As with singular knots, to every singular string link there corresponds a chord diagram which records the combinatorial information of the order of occurence of the double points in the string link. To every T4T or T1T relation there corresponds a (combinatorial) 4T or 1T relation, obtained by replacing each singular string link with its associated chord diagram. Two T4T or T1T relations are said to have the same configuration class if their associated 4T or 1T relations are the same.

In order to make sense of the above Theorem, it is necessary to understand how an arbitrary T4T or T1T relation can be considered a relation among the elements of $S$, since these may be chosen in a completely different way from the relations. In the case of those string links $L \in S$ which have $n$ singularities, and the relations of order $n$, there is no problem. If $L$ and $L'$ have the same chord diagram with $n$ chords, then $[L] = [L'] \in V_n$. Hence the T4T and T1T relations among singular string links become 4T and 1T relations among chord diagrams. (These are the "top row" relations of Vassiliev [11].)

Now suppose we have a T4T or a T1T relation of order $k < n$ Suppose $L$ is string link in the relation. Then there exists an $L' \in S$ such that $L$ and $L'$ share the same chord diagram. It is then possible to make crossing changes to $L$ until it is equivalent to $L'$. Thus our relation of order $k$ becomes a relation among elements of $S$ of order $k$, *plus* a sum of singular string links of order $> k$. Each of these higher-order string links may in turn be written as a sum elements of $S$ plus higher-order singular string links. Inductively, we see that each T4T or T1T relation becomes a relation among the elements of $S$. But there is no reason that the relations should be homogeneous with respect to the degree of the elements of $S$.

(Part of the content of the Theorem is that it does not matter what sequence of crossing changes you choose to transform a singular string link $L$ to $L' \in S$. More specifically, different choices of crossing change sequences will produce relations which differ by higher-order T4T relations.)

Let $A_n = A_n(k)$ be the abelian group generated by all chord diagrams of order $n$, subject to the 4T and 1T relations. We see that there is a homomorphism $\phi : A_n \to V_n$, where if $D$ is a chord diagram of order $n$ then $\phi(D)$ is any singular string link of order $n$ whose chord diagram is $D$. The combinatorial 4T and 1T relations are easier to work with than their topological counterparts, and it would be nice if the map $\phi$ were always injective, indeed this injectivity question goes back to Vassiliev [11]. The Kontsevich integral gives an almost satisfactory answer to this question. It works as well for string links in general as it does for knots, and the result may be stated as follows:

**Theorem** (Kontsevich) *Let $\phi : A_n(k) \to V_n(k)$ be as above. Then the kernel of $\phi$ is finite.*

The existence of torsion in $A_n(1)$ (the case of knots) is unknown. For the case of two-strand string links, the element $\Delta w$, shown below, was found by Dogolazky and Kneissler (see [4]) to have order 2 in $A_5(2)$:

$\Delta w =$ 

Using modified versions of the same computer programs I used to make the tables at the end of this paper, I have found that $\Delta w$ is in the kernel of $\phi$. In fact, it is killed by T4T relations of order 4. Thus there is no analog of the Kontsevich integral over $\mathbb{Z}/2\mathbb{Z}$, at least for string links.

Here is a brief description of the computer programs used to obtain this result. There are two main programs. The first program takes an abitrary singular string link $L$ and makes crossing changes so that it is equivalent to a standard $L'$ with the same chord diagram. In fact, the set $S$ of the standard links $L'$ is not kept explicitly. Rather, there is a process of moves and crossing changes to $L$ which results in a "canonical" $L'$. The process is a slight modification of the algorithm described in [9]. First, a spanning tree is chosen for $L$, where $L$ is viewed here as a spatial graph with four-valent rigid vertices. (The endpoints of the string link may also be treated as edges adjacent to a rigid vertex which is the boundary of $\mathbb{D}^2 \times [0,1]$.) Also, a cyclic orientation is chosen for the edges at each vertex, compatible with the dihedral orientation required by the rigidness of the vertex. The choices of spanning tree and cyclic orientation are done by a simple but arbitrary algorithm, whose only important property is that it makes the same choices for any two singular string links with the same chord diagram.

After the spanning tree and orientation are chosen, Reidemeister moves are performed until the spanning tree has no crossings on it, and until all the cyclic orderings on all the vertices match those chosen above. Then the remaining edges of the spatial graph $L$ are layered. Each time a crossing change is made, the new singular string link (with one more singularity than $L$) is inductively processed by the same algorithm until the number of singularities exceeds the order of the invariant to be computed.

The second program used in these computations is a generator of T4T relations. A list of the configuration classes of 4T relations is generated, and then the program realizes each one as a T4T relation. As noted above, it does not matter which particular T4T relation is chosen, since any two T4T relations of the same configuration class will differ by T4T relations among string links with more singularities.

After the T4T relations are generated, they are fed into the first program, and turned into linear combinations among the singular string links in the chosen set $S$ (which exists only implicitly, as noted above). The result is a list of linear equations with the variables indexed by the chord diagrams with $\leq n$ chords. The equations are solved, and a basis chosen. Thereafter, in order to compute the invariants of a give string link $L$, only the first program is necessary.

Unfortunately, there doesn't seem to be an easy way to extract from the data files an understandable linear combination of T4T relations which adds up to $\Delta w$. It would be nice to have an understanding of how exactly the 2-torsion is killed.

# 4 Knot tables

First we give matrices to translate our invariants into finite-type the invariants obtained from standard knot polynomials, following Kanenobu [7].

The HOMFLYPT polynomial of a knot $K$ is written

$$P(K;t,z) = \sum_{i=0}^{N} P_{2i}(K;t)z^{2i}$$

where $P_{2i}(K,t) \in \mathbb{Z}[t^{\pm t}]$, and is determined by the skein relation

$$t^{-1}P(L_+;t,z) - tP(L_-;t,z) = zP(L_0;t,z)$$

The Jones polynomial $V(L;t)$ is given by

$$V(L;t) = P(L;t,t^{1/2} - t^{-1/2})$$

The Conway polynomial is given by

$$\Delta_K(z) = P(K;1,z)$$

and is written

$$\Delta_K(z) = \sum_{i=0}^{N} a_{2i}(K)z^{2i}$$

The Kauffman polynomial of a knot $K$ is written

$$F(K;a,z) = \sum_{i=0}^{N} F_i(K;a)z^i$$

where $F_i(K;a) \in \mathbb{Z}[a^{\pm 1}]$, and is determined by the skein relation

$$aP(L;a,z) + a^{-1}P(L_-;a,z) = z(F(L_0;a,z) + a^{-2\nu}F(L_\infty;a,z))$$

The notation $P_{2i}^{(n)}$ denotes the knot invariant obtained by evaluating the $n$th derivative of the polynomial $P_{2i}K;t$ at $t=1$, and similarly for the polynomials $V$ and $F_i$.

$$\left[v_2, v_3, v_{4a}, v_{4b}, v_2^2, v_{5a}, v_{5b}, v_{5c}, v_2v_3, v_{6a}, v_{6b}, v_{6c}, v_{6d}, v_{6e}, v_2^3, v_3^2, v_2v_{4a}, v_2v_{4b}\right] M_1$$

$$= \left[a_2, a_4, \frac{P_0^{(3)}}{24}, \frac{P_0^{(4)}}{24}, a_2^3, a_2a_4, \frac{a_2P_0^{(4)}}{24}, (\frac{P_0^{(3)}}{24})^2, \frac{V^{(5)}}{5!}, \frac{V^{(6)}}{6!}\right]$$

where

$$M_1 = \begin{bmatrix} 1 & -\frac{3}{2} & 1 & -93 & 0 & 0 & 0 & 0 & 128 & -\frac{5327}{2} \\ 0 & 0 & 2 & -12 & 0 & 0 & 0 & 0 & 275 & -\frac{1345}{2} \\ 0 & 1 & 0 & 24 & 0 & 0 & 0 & 0 & -30 & \frac{1177}{2} \\ 0 & 0 & 0 & -16 & 0 & 0 & 0 & 0 & 24 & -538 \\ 0 & \frac{1}{2} & 0 & 8 & 0 & -\frac{3}{2} & -93 & 1 & -9 & 201 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -54 & 135 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 138 & -345 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & -75 \\ 0 & 0 & 0 & 0 & 0 & 0 & -12 & 4 & -18 & 45 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -81 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 27 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 21 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -270 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 156 \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{2} & 8 & 0 & 0 & -\frac{9}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 18 \\ 0 & 0 & 0 & 0 & 0 & 1 & 24 & 0 & 0 & -45 \\ 0 & 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 & 36 \end{bmatrix}$$

and

$$\left[v_2, v_3, v_{4a}, v_{4b}, v_2^2, v_{5a}, v_{5b}, v_{5c}, v_2 v_3, v_{6a}, v_{6b}, v_{6c}, v_{6d}, v_{6e}, v_2^3, v_3^2, v_2 v_{4a}, v_2 v_{4b}\right] M_2$$

$$= \left[\frac{P_0^{(6)}}{6!}, \frac{P_2^{(4)}}{4!}, \frac{P_4^{(2)}}{2}, a_6, \frac{F_0^{(6)}}{6!}, \frac{F_1^{(5)}}{5!}, \frac{F_2^{(4)}}{4!}, \frac{F_3^{(3)}}{3!}, \frac{F_4^{(2)}}{2}, F_5^{(1)}\right]$$

where

$$M_2 = \begin{bmatrix} -\frac{14065}{3} & 2307 & -348 & \frac{40}{3} & \frac{14065}{3} & 9697 & \frac{12413}{2} & \frac{4478}{3} & 120 & 4 \\ -1075 & \frac{533}{2} & -9 & 0 & -1075 & -1524 & -\frac{1133}{2} & -56 & -1 & 0 \\ 1044 & -\frac{1017}{2} & 72 & -2 & -1044 & -2136 & -\frac{2719}{2} & -335 & -20 & 4 \\ -888 & 389 & -48 & 1 & 888 & 1910 & 1375 & 425 & 56 & 2 \\ 380 & -205 & 34 & -\frac{3}{2} & -380 & -706 & -\frac{917}{2} & -116 & -8 & 0 \\ 240 & -72 & 3 & 0 & 240 & 320 & 84 & 0 & 0 & 0 \\ -560 & 144 & -5 & 0 & -560 & -800 & -288 & -20 & 1 & 0 \\ -160 & 60 & -3 & 0 & -160 & -192 & -12 & 16 & 1 & 0 \\ 80 & -24 & 1 & 0 & 80 & 96 & 36 & 4 & 0 & 0 \\ -160 & 96 & -18 & 1 & 160 & 320 & 168 & 24 & 0 & 0 \\ 32 & 0 & -6 & 1 & -32 & -80 & -96 & -68 & -18 & 0 \\ 64 & -56 & 14 & -1 & -64 & -96 & 8 & 24 & -2 & -2 \\ -512 & 288 & -48 & 2 & 512 & 1056 & 608 & 104 & 0 & 0 \\ 256 & -112 & 12 & 0 & -256 & -576 & -432 & -136 & -20 & -2 \\ -\frac{32}{3} & 8 & -2 & \frac{1}{6} & \frac{32}{3} & 16 & 8 & \frac{4}{3} & 0 & 0 \\ 32 & -16 & 2 & 0 & -32 & -64 & -48 & -16 & -2 & 0 \\ -96 & 64 & -14 & 1 & 96 & 176 & 88 & 4 & -4 & 0 \\ 64 & -32 & 4 & 0 & -64 & -128 & -96 & -32 & -4 & 0 \end{bmatrix}$$

The notation used in the tables is as follows. As above, $v_2$ is an invariant of order 2, $v_{4a}$ and $v_{4b}$ are invariants of order 4, and so forth. Odd-order invariants in the tables change sign under mirror image, and even-order invariants are unchanged under mirror image.

The numbering of the knots follows Rolfsen [8], up to mirror images. The invariants were computed directly from the braid words listed, which were obtained from Jones [6]. The knots 10.167 and 10.170 are switched with respect to [6], but not with respect to [8]. In each braid word, $a = \sigma_1$ (a positive crossing between the first and second braid strands), $A = \sigma_1^{-1}$, $b = \sigma_2$, $B = \sigma_2^{-1}$, and so forth.

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# Tables

| Knot | Braid word | $v_2$ | $v_3$ | $v_{4a}$ | $v_{4b}$ | $v_{5a}$ | $v_{5b}$ | $v_{5c}$ | $v_{6a}$ | $v_{6b}$ | $v_{6c}$ | $v_{6d}$ | $v_{6e}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 03.001 | aaa | 1 | -1 | 1 | -3 | -3 | 1 | -2 | 3 | -3 | 2 | -3 | -1 |
| 04.001 | aBaB | -1 | 0 | -2 | 3 | 0 | 0 | 0 | -6 | 4 | -4 | 2 | 1 |
| 05.001 | aaaaa | 3 | -5 | 1 | -6 | -12 | 4 | -8 | -6 | -6 | 0 | -7 | 2 |
| 05.002 | aabbAb | 2 | -3 | 1 | -5 | -7 | 3 | -5 | 1 | -5 | 1 | -6 | 0 |
| 06.001 | AbAcBcb | -2 | 1 | -5 | 5 | 4 | -1 | 2 | -10 | 7 | -8 | 2 | 0 |
| 06.002 | AbAbbb | -1 | 1 | -3 | 1 | 3 | -1 | 1 | -1 | 3 | -3 | 0 | -3 |
| 06.003 | AbbAAb | 1 | 0 | 2 | -2 | 0 | 0 | 0 | 0 | -4 | 2 | -1 | 1 |
| 07.001 | aaaaaaa | 6 | -14 | -4 | -3 | -21 | 7 | -14 | -33 | -7 | -8 | -5 | 8 |
| 07.002 | AcccbaaCb | 3 | -6 | 0 | -5 | -9 | 6 | -7 | -3 | -8 | -3 | -9 | 4 |
| 07.003 | aabAbbbb | 5 | -11 | -3 | -6 | -16 | 8 | -13 | -18 | -8 | -9 | -9 | 7 |
| 07.004 | aabccAbCb | 4 | -8 | -2 | -8 | -10 | 8 | -10 | 0 | -8 | -6 | -13 | 3 |
| 07.005 | aaaabAbb | 4 | -8 | 0 | -5 | -14 | 6 | -9 | -12 | -8 | -3 | -9 | 5 |
| 07.006 | aBAAcbbbc | 1 | -2 | 0 | -3 | -2 | 3 | -2 | 4 | -3 | 0 | -5 | -1 |
| 07.007 | aCbCbAbCb | -1 | 1 | -1 | 4 | 0 | -2 | 0 | -8 | 4 | -4 | 4 | 3 |
| 08.001 | AbcBAddcbD | -3 | 3 | -9 | 5 | 12 | -3 | 5 | -6 | 5 | -9 | 0 | -4 |
| 08.002 | AbbbbbAb | 0 | 1 | -3 | -6 | 2 | 0 | -3 | 8 | -4 | 1 | -2 | -7 |
| 08.003 | AABaddcDBc | -4 | 0 | -14 | 8 | 0 | 0 | 0 | -24 | 8 | -18 | -3 | -3 |
| 08.004 | aaacBCCaB | -3 | 1 | -11 | 4 | 0 | -2 | -1 | -14 | 6 | -11 | -2 | -6 |
| 08.005 | aaaBaaaB | -1 | 3 | -5 | -5 | 5 | -3 | -2 | 3 | -3 | 0 | 2 | -10 |
| 08.006 | AbACbbbcc | -2 | 3 | -7 | 0 | 9 | -3 | 2 | 1 | 1 | -4 | 0 | -8 |
| 08.007 | aaaaBBaB | 2 | -2 | 4 | -2 | -7 | 1 | -3 | -9 | -7 | 1 | -1 | 4 |
| 08.008 | AbaaCbbCC | 2 | -1 | 3 | -4 | -2 | 1 | -1 | -4 | -7 | 1 | -3 | 1 |
| 08.009 | AbAAAbbb | -2 | 0 | -8 | 1 | 0 | 0 | 0 | -6 | 4 | -6 | -2 | -7 |
| 08.010 | AbbAAbbb | 3 | -3 | 3 | -6 | -5 | 3 | -3 | -3 | -9 | 1 | -7 | 1 |
| 08.011 | AbbCbccAb | -1 | 2 | -4 | -2 | 8 | -1 | 2 | 10 | -1 | 0 | -2 | -7 |
| 08.012 | aBcDcDbaCB | -3 | 0 | -8 | 8 | 0 | 0 | 0 | -16 | 10 | -12 | 2 | 2 |
| 08.013 | aabCbACCb | 1 | -1 | 3 | 0 | -6 | 0 | -3 | -10 | -4 | -1 | 2 | 4 |
| 08.014 | aabbACbCb | 0 | 0 | -2 | -3 | -2 | 0 | -3 | 4 | 0 | 1 | -2 | -4 |
| 08.015 | aaBaccbbc | 4 | -7 | 1 | -7 | -16 | 5 | -10 | -8 | -7 | 2 | -9 | 3 |
| 08.016 | aaBaaBaB | 1 | -1 | 3 | 0 | 2 | 2 | 2 | -4 | -4 | -2 | -1 | 2 |
| 08.017 | AbAbbAAb | -1 | 0 | -4 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | -1 | -3 |
| 08.018 | aBaBaBaB | 1 | 0 | 0 | -5 | 0 | 0 | 0 | 14 | -4 | 8 | -5 | -2 |
| 08.019 | abababba | 5 | -10 | 0 | -5 | -18 | 6 | -10 | -14 | -8 | 0 | -11 | 4 |
| 08.020 | aaabAAAb | 2 | -2 | 2 | -5 | -1 | 3 | -1 | 5 | -7 | 3 | -6 | 0 |
| 08.021 | aBBaabbb | 0 | 1 | -1 | -3 | 1 | -1 | -1 | 5 | -1 | 1 | -1 | -5 |
| 09.001 | aaaaaaaaa | 10 | -30 | -20 | 15 | -9 | 3 | -6 | -27 | -13 | 0 | 21 | -12 |
| 09.002 | abcdddcDbaCB | 4 | -10 | -2 | -2 | -5 | 10 | -5 | 1 | -16 | -9 | -14 | 10 |
| 09.003 | aBaaaaaabb | 9 | -26 | -18 | 6 | -7 | 9 | -12 | -23 | -15 | -18 | 11 | 5 |
| 09.004 | AcbaabccccB | 7 | -19 | -11 | -1 | -5 | 13 | -11 | -5 | -19 | -21 | -9 | 12 |
| 09.005 | aabAcBcddcDb | 6 | -15 | -9 | -7 | 0 | 15 | -10 | 20 | -17 | -18 | -20 | 10 |
| 09.006 | aabbaaaaaB | 7 | -18 | -6 | 3 | -17 | 7 | -8 | -25 | -11 | -4 | -8 | 3 |
| 09.007 | aaabccAbbbC | 5 | -12 | -2 | -1 | -9 | 9 | -5 | -5 | -15 | -7 | -15 | 7 |
| 09.008 | aBcAADcBccdc | 0 | -2 | -2 | -1 | 1 | 4 | 0 | 3 | 0 | -2 | -4 | -2 |
| 09.009 | aaaBaaaabb | 8 | -22 | -12 | 4 | -11 | 9 | -10 | -23 | -15 | -14 | -1 | 5 |
| 09.010 | AbaabbbCbcc | 8 | -22 | -16 | -2 | -2 | 14 | -14 | 0 | -16 | -26 | -4 | 12 |
| 09.011 | AbCbAbbbbcb | 4 | -9 | -3 | -7 | -6 | 10 | -8 | 4 | -10 | -8 | -15 | 4 |
| 09.012 | aBAAcbbbddcD | 1 | -3 | -1 | -2 | 4 | 6 | 1 | 10 | -6 | -3 | -9 | 1 |
| 09.013 | aabCbAccbbb | 7 | -18 | -10 | -3 | -8 | 12 | -12 | -8 | -14 | -18 | -10 | 9 |
| 09.014 | addCbCbCADbbcB | -1 | 2 | 0 | 4 | -4 | -5 | -2 | -12 | 5 | -4 | 7 | 2 |
| 09.015 | aBacBdCddc | 2 | -5 | -1 | -4 | -3 | 7 | -4 | 5 | -7 | -4 | -10 | 2 |
| 09.016 | aaBaaabbbb | 6 | -14 | -2 | 0 | -12 | 8 | -4 | -2 | -14 | -2 | -19 | 1 |
| 09.017 | aaacBaBCBaB | -2 | 0 | -4 | 7 | 5 | 1 | 4 | -9 | 9 | -10 | 1 | 2 |
| 09.018 | aaCbAbbccbb | 6 | -15 | -5 | -1 | -14 | 9 | -10 | -20 | -13 | -10 | -9 | 9 |

| Knot | Braid word | $v_2$ | $v_3$ | $v_{4a}$ | $v_{4b}$ | $v_{5a}$ | $v_{5b}$ | $v_{5c}$ | $v_{6a}$ | $v_{6b}$ | $v_{6c}$ | $v_{6d}$ | $v_{6e}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 09.019 | aabACDCbCbdC | -2 | -1 | -3 | 8 | 4 | 3 | 3 | -16 | 9 | -11 | 5 | 5 |
| 09.020 | abbCbAbCbbb | 2 | -4 | 0 | -4 | 0 | 6 | -1 | 8 | -8 | -3 | -11 | 0 |
| 09.021 | AAcDcBacddbb | 3 | -6 | -2 | -8 | 0 | 9 | -4 | 20 | -9 | -2 | -17 | 0 |
| 09.022 | aBcccBcABcB | -1 | -1 | -1 | 5 | 5 | 3 | 4 | -7 | 5 | -8 | 1 | 2 |
| 09.023 | aBaabcccbCb | 5 | -11 | -1 | -3 | -15 | 7 | -8 | -15 | -9 | -4 | -12 | 4 |
| 09.024 | accBacBBB | 1 | -2 | 0 | -4 | 2 | 4 | 1 | 8 | -4 | 3 | -5 | -2 |
| 09.025 | AbADCbddcddbbC | 0 | -1 | -3 | -2 | 0 | 2 | -2 | 0 | 0 | -4 | -4 | -3 |
| 09.026 | AbbbcbAbACb | 0 | 1 | 1 | 0 | -5 | -3 | -3 | -3 | 1 | 1 | 3 | 0 |
| 09.027 | AbAACbAbbcb | 0 | -1 | -1 | -1 | 1 | 2 | 1 | -1 | 0 | -1 | -1 | -2 |
| 09.028 | aaccBBacB | 1 | 0 | 2 | -3 | -1 | 0 | -1 | 5 | -4 | 3 | -2 | -1 |
| 09.029 | AbCbAbCbb | 1 | -2 | 2 | -1 | 10 | 6 | 6 | 0 | -4 | -4 | -4 | -1 |
| 09.030 | aCCbCAbaaCb | -1 | -1 | -3 | 2 | -3 | 1 | -1 | -5 | 3 | -3 | 1 | 0 |
| 09.031 | AbbCbbAbcAb | 2 | -2 | 2 | -5 | -9 | 1 | -6 | 1 | -5 | 4 | -4 | 1 |
| 09.032 | AbcAbAccbCb | -1 | 2 | -2 | 1 | 2 | -3 | 0 | 0 | 3 | -2 | 2 | -3 |
| 09.033 | AAAbAbbcaBc | 1 | -1 | 1 | -3 | 5 | 3 | 3 | 7 | -5 | 5 | -3 | 1 |
| 09.034 | aBcBaBcaB | -1 | 0 | -2 | 3 | 8 | 2 | 5 | -2 | 2 | -1 | 2 | 3 |
| 09.035 | accDBabccbCdCb | 7 | -18 | -14 | -9 | 6 | 18 | -12 | 38 | -16 | -24 | -22 | 11 |
| 09.036 | AbbbCbccAbc | 3 | -7 | -1 | -4 | -9 | 7 | -7 | -9 | -7 | -7 | -9 | 4 |
| 09.037 | aBcBcAdCBdcB | -3 | -1 | -7 | 10 | 0 | 2 | 1 | -20 | 12 | -15 | 3 | 5 |
| 09.038 | aabccbbAbCb | 6 | -14 | -4 | -3 | -13 | 9 | -9 | -9 | -13 | -5 | -13 | 7 |
| 09.039 | ACbdccAbbcdB | 2 | -4 | -2 | -7 | 1 | 7 | -3 | 19 | -7 | -1 | -14 | -2 |
| 09.040 | acBcaBacB | -1 | 1 | -3 | 1 | -5 | -3 | -4 | -5 | 5 | -2 | 2 | -2 |
| 09.041 | AbbDcDBaCbcDcb | 0 | 1 | 3 | 3 | -11 | -5 | -5 | -23 | 3 | -5 | 9 | 3 |
| 09.042 | aaacBcAAB | -2 | 0 | -6 | 4 | 0 | 0 | 0 | -6 | 6 | -6 | 0 | -1 |
| 09.043 | abaabbcBaBC | 1 | -2 | -2 | -5 | -7 | 2 | -7 | -3 | -2 | -1 | -3 | -1 |
| 09.044 | AbAcBcbbC | 0 | -1 | 1 | 2 | 1 | 2 | 1 | -7 | 0 | -3 | 2 | 3 |
| 09.045 | aBacbbbcB | 2 | -4 | 0 | -4 | -8 | 4 | -6 | -6 | -4 | -4 | -6 | 1 |
| 09.046 | acbACbacB | -2 | 3 | -5 | 3 | 11 | -3 | 5 | 3 | 3 | -3 | 1 | -3 |
| 09.047 | AbcAbAbcb | -1 | 2 | 0 | 4 | 4 | -3 | 3 | -6 | 5 | -5 | 4 | 0 |
| 09.048 | aaBcbbACbCb | 3 | -5 | -1 | -9 | -3 | 7 | -5 | 17 | -7 | 1 | -15 | -2 |
| 09.049 | aabbcbAbbcB | 6 | -14 | -6 | -6 | -14 | 10 | -14 | -14 | -10 | -12 | -10 | 9 |
| 10.001 | aaDECbAcdEdcb | -4 | 6 | -14 | 2 | 23 | -6 | 7 | 11 | -8 | -3 | -2 | -10 |
| 10.002 | aBaaaaaaaB | 2 | -2 | -4 | -15 | -7 | 5 | -16 | -9 | -15 | -4 | 4 | 5 |
| 10.003 | aCBaEEbcDecbD | -6 | 3 | -27 | 7 | 15 | -3 | 5 | -29 | -13 | -21 | -16 | -10 |
| 10.004 | aaDDDbcBADcb | -5 | 1 | -23 | 4 | 16 | 2 | 8 | -22 | -8 | -14 | -12 | -11 |
| 10.005 | AAbAbbbbbb | 4 | -7 | 5 | 0 | -20 | 2 | -7 | -32 | -8 | -3 | -4 | 5 |
| 10.006 | aBcABAcbbbbbb | -1 | 4 | -8 | -12 | 7 | -2 | -8 | 5 | -16 | 2 | 7 | -10 |
| 10.007 | aCBabcDcDccb | -1 | 3 | -5 | -6 | 14 | 0 | 2 | 28 | -10 | 6 | -3 | -8 |
| 10.008 | aaaaacBaCCB | -3 | 4 | -16 | -6 | 6 | -5 | -5 | -8 | -11 | -1 | 0 | -16 |
| 10.009 | AAAbAbbbbb | -2 | 2 | -12 | -6 | 7 | -1 | -1 | -1 | -7 | 1 | -2 | -13 |
| 10.010 | ACdccbaaCbDCbC | 1 | -2 | 4 | 3 | -14 | -1 | -6 | -30 | -1 | -8 | 6 | 6 |
| 10.011 | AACBadcccBcd | -5 | 4 | -24 | 1 | 9 | -6 | -1 | -25 | -14 | -13 | -9 | -16 |
| 10.012 | AAbAccbbbbC | 4 | -6 | 4 | -3 | -15 | 3 | -7 | -33 | -11 | -7 | -3 | 5 |
| 10.013 | aaCbACdeedcEBBD | -5 | 2 | -18 | 10 | 6 | -3 | 2 | -22 | 5 | -18 | -6 | -1 |
| 10.014 | AbAccbbbbCb | 2 | -3 | -3 | -11 | -10 | 4 | -13 | 0 | -6 | 1 | -3 | 1 |
| 10.015 | AcBccccBAAb | 3 | -2 | 6 | -2 | -10 | 0 | -5 | -28 | -12 | -5 | 1 | 6 |
| 10.016 | aBAAcBDcddcc | -4 | 4 | -18 | 0 | 15 | -4 | 3 | -7 | -8 | -7 | -6 | -14 |
| 10.017 | aaaaBaBBBB | 2 | 0 | 8 | 3 | 0 | 0 | 0 | -24 | -10 | -6 | 4 | 9 |
| 10.018 | accddCBdAAcB | -2 | 1 | -9 | -1 | -6 | -3 | -6 | -2 | 3 | 0 | -1 | -7 |
| 10.019 | AABaccccBcB | 1 | 0 | 6 | 4 | 5 | 1 | 4 | -21 | -5 | -8 | 5 | 7 |
| 10.020 | ACbbbcddcDAb | -3 | 6 | -12 | -4 | 17 | -6 | 1 | 9 | -12 | 1 | 3 | -13 |
| 10.021 | AbAccbCbbbb | 1 | 0 | -4 | -13 | 3 | 4 | -8 | 19 | -14 | 2 | -4 | -4 |
| 10.022 | AAAbccAbbbC | -4 | 2 | -20 | -1 | 11 | -1 | 3 | -17 | -9 | -9 | -8 | -16 |
| 10.023 | AbAbbbCbAcc | 3 | -5 | 5 | 0 | -19 | 1 | -8 | -35 | -7 | -6 | 1 | 6 |
| 10.024 | AbCbbaabcDcD | -2 | 5 | -9 | -7 | 17 | -3 | 1 | 21 | -13 | 3 | 0 | -14 |
| 10.025 | AccbbbCbbAb | 0 | 2 | -4 | -10 | 7 | 1 | -4 | 17 | -13 | 4 | 0 | -6 |

| Knot | Braid word | $v_2$ | $v_3$ | $v_{4a}$ | $v_{4b}$ | $v_{5a}$ | $v_{5b}$ | $v_{5c}$ | $v_{6a}$ | $v_{6b}$ | $v_{6c}$ | $v_{6d}$ | $v_{6e}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10.026 | abbCBBBaBaacB | -3 | 2 | -14 | -1 | 14 | 0 | 5 | -2 | -4 | -3 | -5 | -11 |
| 10.027 | AAbCbccccAb | 2 | -3 | 5 | 1 | -14 | 0 | -5 | -22 | -4 | -3 | 1 | 5 |
| 10.028 | aabCbADcDDcb | 3 | -4 | 4 | -3 | -13 | 2 | -7 | -27 | -10 | -7 | -1 | 4 |
| 10.029 | aBcDcBABcBdccc | -4 | 3 | -15 | 5 | 5 | -5 | 0 | -13 | 3 | -10 | -3 | -6 |
| 10.030 | aabCbAccdCbCDb | 1 | -1 | -3 | -9 | -1 | 3 | -6 | 17 | -5 | 4 | -7 | -5 |
| 10.031 | aabbAdCDDCbC | 2 | 1 | 5 | -1 | 10 | 1 | 6 | -22 | -9 | -6 | 2 | 4 |
| 10.032 | AbCbaaCCCbb | -1 | 0 | -6 | -3 | -5 | -1 | -4 | 3 | 1 | 2 | -2 | -7 |
| 10.033 | aBcDcBAABcdd | 0 | 0 | 4 | 6 | 0 | 0 | 0 | -24 | 0 | -10 | 8 | 8 |
| 10.034 | ADDccbaaDCbc | 3 | -3 | 3 | -5 | -4 | 3 | -2 | -12 | -9 | -4 | -6 | 1 |
| 10.035 | AbCbCdEdEaBcDbb | -4 | 2 | -12 | 9 | 9 | -2 | 4 | -11 | 8 | -12 | -1 | 1 |
| 10.036 | aaCbbcDcDAcb | 1 | -2 | -2 | -6 | -9 | 2 | -9 | 1 | -2 | 3 | -2 | 0 |
| 10.037 | adCDDbCCbaaB | 3 | 0 | 4 | -5 | 0 | 0 | 0 | -18 | -10 | -4 | -3 | 2 |
| 10.038 | aadCBBcccddCAb | -1 | 2 | -6 | -5 | 1 | -2 | -5 | 1 | -2 | 1 | 2 | -7 |
| 10.039 | AbAbbbCbccc | 1 | -1 | -3 | -9 | -6 | 2 | -10 | -4 | -6 | 0 | 1 | -1 |
| 10.040 | aabAbbCbaCC | 3 | -4 | 4 | -3 | -8 | 3 | -3 | -10 | -9 | -1 | -6 | 3 |
| 10.041 | aBcBADcBcdcBcc | -2 | 2 | -6 | 2 | 10 | -1 | 4 | 6 | 3 | -2 | -1 | -4 |
| 10.042 | aBAAcDbbcDDc | 0 | -1 | 1 | 2 | -7 | 0 | -4 | -13 | 0 | -6 | 3 | 2 |
| 10.043 | AbCDbCCabCDcbb | 2 | 0 | 2 | -5 | 0 | 0 | 0 | -4 | -6 | -2 | -5 | -2 |
| 10.044 | ABcBcddabCBBdc | 0 | -1 | -1 | -1 | -4 | 1 | -3 | 4 | 1 | 3 | -1 | 0 |
| 10.045 | aCbCbdCAbCbDCb | -2 | 0 | -4 | 7 | 0 | 0 | 0 | -18 | 8 | -12 | 4 | 3 |
| 10.046 | AbbbbbAbbb | 0 | 4 | -6 | -18 | -2 | -2 | -17 | -18 | -20 | 1 | 20 | -5 |
| 10.047 | aaaaaBaaBB | 6 | -11 | -1 | -6 | -10 | 8 | -7 | -16 | -12 | -9 | -14 | 3 |
| 10.048 | AAbbbbAAAb | 4 | 0 | 6 | -5 | -11 | -3 | -6 | -21 | -15 | -4 | -5 | 4 |
| 10.049 | aaaacbbbcaB | 7 | -16 | -4 | -3 | -23 | 7 | -13 | -21 | -7 | 3 | -10 | 4 |
| 10.050 | AbbCbccAbbb | -1 | 5 | -7 | -13 | 9 | -3 | -7 | 7 | -17 | 3 | 9 | -13 |
| 10.051 | aabCCAbbCbb | 5 | -8 | 0 | -8 | -5 | 8 | -5 | -5 | -12 | -7 | -15 | 2 |
| 10.052 | AABccBcccaB | 3 | -1 | 5 | -4 | 10 | 4 | 6 | -14 | -12 | -6 | -5 | 1 |
| 10.053 | aBccdbAccBcAddbb | 6 | -13 | -3 | -6 | -19 | 8 | -13 | -15 | -8 | -1 | -11 | 6 |
| 10.054 | AABccBcccAb | 4 | -2 | 4 | -7 | -2 | 2 | -1 | -16 | -12 | -3 | -7 | 2 |
| 10.055 | AdcdbaaabccDcB | 5 | -10 | 0 | -6 | -17 | 7 | -10 | -7 | -9 | 2 | -13 | 4 |
| 10.056 | AbbccBcAbbb | 0 | 2 | -4 | -10 | -4 | -2 | -10 | -4 | -6 | 4 | 7 | -6 |
| 10.057 | aaabCCAbbCb | 4 | -6 | 2 | -6 | -6 | 6 | -4 | -2 | -12 | -2 | -12 | 3 |
| 10.058 | abEdCbCDEaBcd | -4 | 1 | -11 | 11 | 5 | -1 | 3 | -21 | 13 | -17 | 1 | 3 |
| 10.059 | abbdAACdCBcddC | -1 | 1 | -3 | 0 | 2 | -1 | 0 | 4 | 3 | 0 | 0 | -4 |
| 10.060 | AAbbaBcDcBcBdc | -1 | 1 | -1 | 3 | 4 | -1 | 3 | -4 | 3 | 1 | 5 | 3 |
| 10.061 | aBAAcccBccc | -4 | 5 | -21 | -5 | 1 | -9 | -9 | -29 | -15 | -5 | 2 | -18 |
| 10.062 | AAbbbAbbbb | 5 | -9 | 3 | -2 | -15 | 5 | -6 | -29 | -13 | -6 | -8 | 5 |
| 10.063 | aabccAdCBcddcb | 6 | -14 | -4 | -4 | -20 | 8 | -14 | -24 | -8 | -4 | -7 | 9 |
| 10.064 | AAAbbbAbbb | -3 | 3 | -17 | -6 | 4 | -4 | -4 | -18 | -10 | -4 | -1 | -18 |
| 10.065 | acbbcABcccABB | 4 | -7 | 3 | -3 | -13 | 5 | -7 | -23 | -13 | -7 | -6 | 6 |
| 10.066 | aabCbbaBccccb | 7 | -17 | -5 | 0 | -24 | 6 | -13 | -34 | -6 | 1 | -4 | 6 |
| 10.067 | AbDCbDaaccbbdC | 0 | 0 | -4 | -6 | 2 | 2 | -4 | 8 | -4 | 0 | -3 | -5 |
| 10.068 | AbCddaaBcbbDCC | 2 | -3 | 5 | 1 | -19 | -1 | -9 | -39 | -5 | -9 | 6 | 6 |
| 10.069 | acdbCdBBAAcbbb | 2 | -4 | 0 | -4 | -5 | 5 | -5 | 5 | -7 | -3 | -9 | 2 |
| 10.070 | aBcccAAbbDcD | -3 | 2 | -10 | 4 | 7 | -2 | 2 | -1 | 4 | -6 | -2 | -4 |
| 10.071 | aaDDBCbbCDbcAb | 1 | 0 | 2 | -1 | 0 | 0 | 0 | -8 | -4 | -2 | 0 | 2 |
| 10.072 | aaaBaabCbCb | 2 | -4 | -2 | -7 | -15 | 3 | -13 | -13 | -3 | 1 | 0 | 5 |
| 10.073 | aBccaDcBcdcB | 1 | -2 | 0 | -2 | -5 | 2 | -4 | -3 | -2 | -4 | -4 | 0 |
| 10.074 | AdbCbDCbaaDccb | 0 | 2 | -4 | -10 | 12 | 2 | 0 | 38 | -12 | 8 | -8 | -10 |
| 10.075 | aBacBcdcBcDB | 0 | 1 | 1 | 0 | 0 | -2 | 1 | -4 | 0 | 3 | 4 | 1 |
| 10.076 | aaBAcccBaBccc | -2 | 6 | -10 | -10 | 10 | -6 | -6 | 0 | -16 | 4 | 11 | -13 |
| 10.077 | AbbbCCaabbC | 4 | -5 | 3 | -6 | -8 | 4 | -4 | -14 | -10 | -4 | -9 | 1 |
| 10.078 | AAdcBaccDcbbbc | 3 | -5 | 1 | -7 | -10 | 5 | -7 | 2 | -7 | 3 | -9 | 1 |
| 10.079 | aaaBBaaBBB | 5 | 0 | 4 | -9 | 0 | 0 | 0 | -16 | -14 | -2 | -10 | 3 |
| 10.080 | aabCbbAbbbcbb | 6 | -12 | 0 | -6 | -21 | 7 | -11 | -7 | -9 | 5 | -16 | 2 |
| 10.081 | AABBACbddccd | 3 | 0 | 2 | -8 | 0 | 0 | 0 | 8 | -10 | 4 | -10 | 0 |

| Knot | Braid word | $v_2$ | $v_3$ | $v_{4a}$ | $v_{4b}$ | $v_{5a}$ | $v_{5b}$ | $v_{5c}$ | $v_{6a}$ | $v_{6b}$ | $v_{6c}$ | $v_{6d}$ | $v_{6e}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10.082 | aaaaBBaBaB | 0 | 0 | -4 | -6 | 1 | 1 | -2 | 15 | -5 | 10 | -3 | -3 |
| 10.083 | aabCbbCbACb | 1 | -2 | 4 | 3 | 2 | 3 | 4 | -8 | -3 | -8 | -3 | 2 |
| 10.084 | aaabbCbACbC | 2 | -2 | 4 | -2 | 1 | 3 | 2 | -3 | -7 | -4 | -6 | -1 |
| 10.085 | AbbAbAbbbb | 2 | -3 | 5 | 1 | 2 | 4 | 5 | 0 | -6 | -7 | -10 | -2 |
| 10.086 | AbCbCCbaabC | -1 | 1 | -5 | -2 | 6 | 0 | 2 | 14 | -2 | 8 | -2 | -2 |
| 10.087 | aaaCbCAbCbC | 0 | -1 | -3 | -4 | -4 | 1 | -3 | 8 | -1 | 5 | -3 | -4 |
| 10.088 | aaBDcaBcBdCAbC | -1 | 0 | 0 | 6 | 0 | 0 | 0 | -16 | 4 | -8 | 6 | 6 |
| 10.089 | abCbAdbCbdCd | 1 | -3 | -1 | -2 | -4 | 4 | -4 | -4 | -2 | -8 | -6 | -1 |
| 10.090 | aacBBaaBaBcAB | -3 | 1 | -13 | 1 | 3 | -1 | 0 | -5 | -1 | -2 | -3 | -6 |
| 10.091 | aaaBBaBBaB | 2 | 0 | 6 | 0 | -11 | -3 | -6 | -19 | -7 | -8 | 0 | 2 |
| 10.092 | aaabbCbAbCb | 2 | -3 | -1 | -8 | -14 | 2 | -12 | -6 | -4 | 6 | 0 | 4 |
| 10.093 | AAccBcaBccB | 1 | -1 | 5 | 3 | 13 | 5 | 8 | -25 | -3 | -14 | 3 | 2 |
| 10.094 | aaaBaaBBaB | -2 | 2 | -10 | -3 | 1 | -3 | -3 | -3 | -3 | 3 | 1 | -7 |
| 10.095 | AAbCbAbbccb | 3 | -5 | 3 | -3 | -10 | 4 | -5 | -12 | -8 | -5 | -7 | 2 |
| 10.096 | abCdbacBCCdC | -3 | 2 | -8 | 7 | 9 | -2 | 5 | -3 | 6 | -3 | 2 | 4 |
| 10.097 | aaBcDbbACbccDb | 2 | -4 | -4 | -10 | -6 | 6 | -10 | 14 | -4 | 0 | -11 | -2 |
| 10.098 | AbbccbAbbCb | 0 | 3 | -3 | -11 | 5 | -1 | -5 | 19 | -13 | 9 | 3 | -7 |
| 10.099 | AAbAAbbAbb | 4 | 0 | 4 | -8 | 0 | 0 | 0 | -10 | -10 | -4 | -10 | -2 |
| 10.100 | aaaBaaBaaB | 4 | -7 | 3 | -3 | 3 | 9 | 3 | -1 | -15 | -11 | -17 | -1 |
| 10.101 | AcbbadcbbbccdCAB | 7 | -17 | -7 | -3 | -17 | 9 | -13 | -19 | -9 | -5 | -8 | 7 |
| 10.102 | abbCbACbaCC | -2 | 1 | -9 | -1 | 10 | 1 | 4 | 6 | -1 | 2 | -3 | -6 |
| 10.103 | aabbCbbCAbC | 3 | -4 | 4 | -3 | 3 | 6 | 3 | -5 | -12 | -7 | -10 | 0 |
| 10.104 | aaBBBaaBaB | 1 | 0 | 6 | 4 | 11 | 3 | 6 | -19 | -3 | -8 | 4 | 6 |
| 10.105 | ABcBdacBccddCB | -1 | 0 | -4 | 0 | -5 | -1 | -4 | 7 | 3 | 4 | -1 | -1 |
| 10.106 | aaaBBaaBaB | -1 | 1 | -7 | -5 | 4 | 0 | -1 | 8 | -4 | 5 | -2 | -7 |
| 10.107 | adcBccdCBBABcB | 1 | -1 | 3 | 0 | -3 | 1 | -2 | -13 | -5 | -4 | 2 | 3 |
| 10.108 | AAccBaccBcB | 0 | 0 | 4 | 6 | 16 | 4 | 10 | -22 | 2 | -14 | 5 | 4 |
| 10.109 | AAbbAAbbAb | 3 | 0 | 6 | -3 | 0 | 0 | 0 | -14 | -12 | -2 | -3 | 4 |
| 10.110 | aCbCDcbbbaBCDb | -3 | 3 | -11 | 2 | 7 | -4 | 1 | 1 | 2 | -3 | -2 | -7 |
| 10.111 | aabbCbbAbCb | 1 | 0 | -2 | -10 | -12 | -1 | -13 | -10 | -3 | 5 | 6 | -2 |
| 10.112 | aaaBaBaBaB | 2 | -2 | 0 | -8 | -6 | 2 | -5 | 18 | -8 | 15 | -6 | 3 |
| 10.113 | aaabCbAbCbC | 0 | 1 | 1 | 0 | 3 | -1 | 2 | 3 | 1 | -4 | -2 | -5 |
| 10.114 | ABcBcBBaaCbcc | 1 | -1 | -1 | -6 | 0 | 2 | -1 | 24 | -6 | 15 | -6 | 1 |
| 10.115 | AbcDcBAdcdBcBB | 1 | 0 | 4 | 1 | 0 | 0 | 0 | -16 | -6 | -4 | 4 | 5 |
| 10.116 | aaBaBaBaaB | 0 | 0 | -2 | -3 | -5 | -1 | -4 | 7 | -3 | 10 | 1 | 2 |
| 10.117 | AbCbaabCbbC | 2 | -3 | 3 | -2 | -5 | 3 | -2 | -7 | -5 | -6 | -6 | -1 |
| 10.118 | aaBaBBaBaB | 0 | 0 | 2 | 3 | 0 | 0 | 0 | -14 | 4 | -12 | 1 | -2 |
| 10.119 | aabCCbACbCb | -1 | 0 | -4 | 0 | 3 | 1 | 1 | 9 | -1 | 7 | 0 | 2 |
| 10.120 | abbcDacBcAddccbC | 6 | -13 | -1 | -3 | -18 | 7 | -8 | 0 | -13 | 12 | -15 | 7 |
| 10.121 | AbCbCbaabCb | 1 | -2 | 2 | 0 | 0 | 3 | 1 | -6 | -1 | -9 | -5 | -3 |
| 10.122 | aBcBcBABcbAbb | 2 | -2 | 0 | -8 | 2 | 4 | 0 | 24 | -8 | 14 | -9 | 1 |
| 10.123 | aBaBaBaBaB | -2 | 0 | -6 | 4 | 0 | 0 | 0 | -14 | 10 | -16 | -2 | -8 |
| 10.124 | abbbbbabbb | 8 | -20 | -6 | 4 | -18 | 6 | -5 | -4 | -10 | 9 | -17 | -8 |
| 10.125 | ABBBAbbbbb | 3 | 0 | 4 | -6 | -11 | -3 | -6 | -3 | -11 | 4 | -5 | 3 |
| 10.126 | aBBBabbbbb | 5 | -9 | -1 | -8 | -2 | 10 | -4 | 6 | -14 | -4 | -17 | 4 |
| 10.127 | aaaaabAAbb | 1 | 1 | -1 | -11 | -4 | 0 | -9 | 0 | -8 | 1 | 3 | -6 |
| 10.128 | aacbbcbbAcb | 7 | -17 | -5 | 0 | -16 | 8 | -8 | -10 | -12 | 0 | -14 | 2 |
| 10.129 | AAcBBcbbacB | 2 | -1 | 3 | -4 | 9 | 4 | 5 | -1 | -8 | 1 | -4 | 1 |
| 10.130 | AbCCCbaabcc | 4 | -6 | 0 | -9 | 3 | 9 | -1 | 17 | -13 | 1 | -17 | 2 |
| 10.131 | AbcccbaabCC | 0 | 2 | -2 | -7 | 3 | -1 | -3 | 9 | -5 | 1 | 0 | -9 |
| 10.132 | AbcaaaBBBcb | 3 | -5 | 1 | -6 | -1 | 7 | -2 | 9 | -11 | 0 | -12 | 2 |
| 10.133 | AcbccbaaCBaCb | 1 | 0 | 0 | -7 | -5 | 0 | -6 | 5 | -4 | 4 | -1 | -4 |
| 10.134 | abbabbbcbAbbC | 6 | -13 | -1 | -3 | -18 | 7 | -8 | -8 | -9 | 2 | -17 | 0 |
| 10.135 | abbcBAAABcc | 3 | -1 | 3 | -7 | 2 | 2 | 1 | 0 | -10 | 3 | -7 | 1 |
| 10.136 | aabCbAbbCBB | 0 | -1 | -1 | -1 | 4 | 3 | 2 | 8 | -1 | 2 | -3 | -1 |
| 10.137 | aacBaBcADcBcBd | -2 | 2 | -4 | 5 | 4 | -3 | 2 | -8 | 7 | -6 | 4 | 0 |

| Knot | Braid word | $v_2$ | $v_3$ | $v_{4a}$ | $v_{4b}$ | $v_{5a}$ | $v_{5b}$ | $v_{5c}$ | $v_{6a}$ | $v_{6b}$ | $v_{6c}$ | $v_{6d}$ | $v_{6e}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10.138 | aacBaBcdACbCbD | -3 | 2 | -8 | 7 | 9 | -2 | 5 | -11 | 10 | -13 | 0 | -3 |
| 10.139 | aabbbaabab | 9 | -25 | -13 | 9 | -15 | 5 | -7 | -23 | -11 | 1 | 2 | -7 |
| 10.140 | aBaCCCbcccb | 2 | -4 | 2 | -2 | 0 | 6 | 0 | 4 | -10 | 0 | -7 | 5 |
| 10.141 | AAAbaaBaab | -1 | 1 | -5 | -2 | -2 | -2 | -3 | 0 | 2 | -1 | -1 | -7 |
| 10.142 | Acccbcccaab | 8 | -21 | -11 | 1 | -14 | 9 | -12 | -20 | -11 | -10 | -4 | 4 |
| 10.143 | AbbAAbaabb | 3 | -5 | 3 | -3 | -2 | 6 | 0 | 2 | -12 | 0 | -10 | 4 |
| 10.144 | aacbbAbCCaB | -2 | 2 | -8 | 0 | -2 | -4 | -4 | -8 | 4 | -4 | 1 | -7 |
| 10.145 | aabACbacbbc | 5 | -12 | -4 | -4 | -10 | 10 | -10 | -8 | -14 | -12 | -12 | 10 |
| 10.146 | aaBCCbbaBcB | 0 | 0 | 2 | 3 | 8 | 2 | 5 | -12 | 0 | -5 | 4 | 4 |
| 10.147 | ABBccbbaaBACb | -1 | 0 | -4 | 0 | 3 | 1 | 1 | 1 | 3 | -3 | -2 | -5 |
| 10.148 | aabbbAAbAb | 4 | -7 | 1 | -6 | -7 | 7 | -5 | -3 | -11 | -3 | -12 | 3 |
| 10.149 | AAbbbaaabb | 2 | -2 | 0 | -9 | -10 | 2 | -10 | -4 | -6 | 2 | -1 | 0 |
| 10.150 | aabbcbABccB | 1 | -1 | -1 | -6 | -2 | 2 | -4 | 4 | -4 | 0 | -4 | -4 |
| 10.151 | aaCbbacBBCb | 3 | -4 | 2 | -6 | -7 | 4 | -5 | -3 | -8 | -1 | -8 | 1 |
| 10.152 | aabbbaaabb | 7 | -15 | -1 | -3 | -21 | 7 | -8 | 1 | -9 | 8 | -23 | -5 |
| 10.153 | aabCCBBaCbb | 4 | -1 | 3 | -9 | -5 | 0 | -3 | -1 | -12 | 3 | -10 | 2 |
| 10.154 | aBccbbaacbb | 5 | -9 | 1 | -8 | -20 | 6 | -12 | -8 | -8 | 4 | -12 | 3 |
| 10.155 | aaabAAbAAb | -2 | 2 | -8 | 0 | 6 | -2 | 1 | -4 | 2 | -5 | -1 | -8 |
| 10.156 | AbcbbcAAbCb | 1 | -1 | 3 | 0 | -9 | -1 | -4 | -13 | -1 | -2 | 2 | 2 |
| 10.157 | AbbaaBaabb | 4 | -8 | -2 | -8 | -18 | 6 | -15 | -16 | -6 | -3 | -5 | 8 |
| 10.158 | aabcABBcBBc | -3 | 1 | -11 | 4 | 8 | 0 | 4 | -10 | 4 | -8 | -2 | -4 |
| 10.159 | aBaBabbbaB | 2 | -3 | 3 | -2 | -8 | 2 | -3 | -8 | -4 | -1 | -4 | 1 |
| 10.160 | abccaaBaBaC | 3 | -6 | -2 | -8 | -8 | 7 | -9 | 4 | -7 | -3 | -11 | 2 |
| 10.161 | Abaabbbaab | 7 | -18 | -8 | 0 | -15 | 9 | -12 | -25 | -13 | -12 | -5 | 9 |
| 10.163 | AAcBccabbcB | -3 | 4 | -12 | 0 | 10 | -5 | 1 | -6 | -1 | -7 | -1 | -12 |
| 10.164 | aBcAbaaBaBc | 1 | -2 | 2 | 0 | -8 | 1 | -4 | -12 | -1 | -4 | 0 | 2 |
| 10.165 | AbCCaBaBacb | 1 | 0 | 4 | 1 | 8 | 2 | 5 | -10 | -6 | -1 | 3 | 6 |
| 10.166 | aabCbAbccAb | 2 | -3 | -1 | -8 | -14 | 2 | -12 | -14 | 0 | -4 | -2 | -3 |