# Fragments of geometric topology from the sixties

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## Contents

# Preface

This book presents some of the main themes in the development of the combinatorial topology of high-dimensional manifolds, which took place roughly during the decade 1960–70 when new ideas and new techniques allowed the discipline to emerge from a long period of lethargy.

The first great results came at the beginning of the decade. I am referring here to the weak Poincaré conjecture and to the uniqueness of the PL and differentiable structures of Euclidean spaces, which follow from the work of J Stallings and E C Zeeman. Part I is devoted to these results, with the exception of the first two sections, which offer a historical picture of the salient questions which kept the topologists busy in those days. It should be note that Smale proved a strong version of the Poincaré conjecture also near the beginning of the decade. Smale's proof (his $h$–cobordism theorem) will not be covered in this book.

The principal theme of the book is the problem of the existence and the uniqueness of triangulations of a topological manifold, which was solved by R Kirby and L Siebenmann towards the end of the decade.

This topic is treated using the "immersion theory machine" due to Haefliger and Poenaru. Using this machine the geometric problem is converted into a bundle lifting problem. The obstructions to lifting are identified and their calculation is carried out by a geometric method which is known as Handle-Straightening.

The treatment of the Kirby–Siebenmann theory occupies the second, the third and the fourth part, and requires the introduction of various other topics such as the theory of microbundles and their classifying spaces and the theory of immersions and submersions, both in the topological and PL contexts.

The fifth part deals with the problem of smoothing PL manifolds, and with related subjects including the group of diffeomorphisms of a differentiable manifold.

The sixth and last part is devoted to the bordism of pseudomanifolds a topic which is connected with the representation of homology classes according to Thom and Steenrod. For the main part it describes some of Sullivan's ideas on topological resolution of singularities.

The monograph is necessarily incomplete and fragmentary, for example the important topics of $h$–cobordism and surgery are only stated and for these the reader will have to consult the bibliography. However the book does aim to present a few of the wide variety of issues which made the decade 1960–70 one of the richest and most exciting periods in the history of manifold topology.

### Acknowledgements

(To be extended)

The short proof of 4.7 in the codimension 3 case, which avoids piping, is hitherto unpublished. It was found by Zeeman in 1966 and it has been clarified for me by Colin Rourke.

The translation of the original Italian version is by Rosa Antolini.


### Note about cross-references

Cross references are of the form Theorem 3.7, which means the theorem in subsection 3.7 (of the current part) or of the form III.3.7 which means the results of subsection 3.7 in part III. In general results are unnumbered where reference to the subsection in which they appear is unambiguous but numbered within that subsection otherwise. For example Corollary 3.7.2 is the second corollary within subsection 3.7.

### Note about inset material

Some of the material is inset and marked with the symbol ▼ at the start and ▲ at the end. This material is either of a harder nature or of side interest to the main theme of the book and can safely be omitted on first reading.

### Notes about bibliographic references and ends of proofs

References to the bibliography are in square brackets, eg [Kan 1955]. Similar looking references given in round brackets eg (Kan 1955) are for attribution and do not refer to the bibliography.

The symbol $\square$ is used to indicate either the end of a proof or that a proof is not given.