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ЗАМЕТКИ

ERRATUM TO: “INFINITESIMALS IN ORDERED VECTOR SPACES”

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In this note, Theorem 1 in the article which is cited in the title is corrected.

We give the following Theorem 1 instead of Theorem 1 on page 21 in [1].

Theorem 1. *Let V be an ordered space. Consider the following conditions:*
(1) *V is almost Archimedean*;
(2) $\lambda({}^*V) \cap V = \{0\}$;
(3) $\lambda({}^*V) \subseteq \eta({}^*V)$;
(4) $\lambda({}^*V) \subseteq \text{o-pns}({}^*V)$;
(5) *V is Archimedean.*
Then (1) $\Leftrightarrow$ (2) *and* (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5).

$\lhd$ (1) $\Leftrightarrow$ (2): It follows from the definition of $\lambda({}^*V)$.
(3) $\Rightarrow$ (4): Just, since $\eta({}^*V) \subseteq \text{o-pns}({}^*V)$.
(4) $\Rightarrow$ (5): It is enough to show

$$\Big[V \ni y \leqslant \frac{1}{n}\,u \in V_+ \quad (\forall\, n \in \mathbb{N} \setminus \{0\})\Big] \Rightarrow [y \leqslant 0].$$

Take a $y \in V$, such that $y \leqslant \frac{1}{n}u \in V_+$ for all $n \in \mathbb{N} \setminus \{0\}$. Fix some $\nu \in {}^*\mathbb{N} \setminus \mathbb{N}$. Then $\frac{1}{\nu}\,u \in \lambda({}^*V) \subseteq \text{o-pns}({}^*V)$. Given $z \in V$ then, by the transfer principle, $\frac{1}{\nu}\,u \leqslant z$ iff $\frac{1}{n}u \leqslant z$ for some $n \in \mathbb{N} \setminus \{0\}$. Therefore,

$$U\Big(\frac{1}{\nu}\,u\Big) = \bigcup_{n \in \mathbb{N}\setminus\{0\}} U_n, \quad \text{where} \quad U_n = U\Big(\frac{1}{n}\,u\Big).$$

By the hypothesis, $\inf_V \big(U\big(\frac{1}{\nu}\,u\big) - L\big(\frac{1}{\nu}\,u\big)\big) = 0$. Hence $\inf_V U\big(\frac{1}{\nu}\,u\big) = \inf_V \big(U\big(\frac{1}{\nu}\,u\big) - 0\big) = 0$, since $0 \in L\big(\frac{1}{\nu}\,u\big)$. Thus

$$\inf_V \bigcup_{n \in \mathbb{N}\setminus\{0\}} U_n = 0. \tag{$*$}$$

Since $y \leqslant \frac{1}{n}\,u \in U_n$ for all $n \in \mathbb{N} \setminus \{0\}$ then it follows from $(*)$ that $y \leqslant 0$, what is required.

(5) $\Rightarrow$ (3): Let $\kappa \in \lambda({}^*V)$. Then $-\frac{1}{n}\,u \leqslant \kappa \leqslant \frac{1}{n}\,u$ for some $u \in V_+$ and all $n \in \mathbb{N} \setminus \{0\}$. In order to show $\kappa \in \eta({}^*V)$, it is sufficient to prove that $\inf_V U(\kappa) = 0$. Take a $w \in U(\kappa)$. Then $-\frac{1}{n}u \leqslant \kappa \leqslant w$ for all $n \in \mathbb{N} \setminus \{0\}$. Since V is Archimedean, $\inf_{n \in \mathbb{N}\setminus\{0\}} \frac{1}{n}\,u = 0$ and

$$0 = -\inf_{n \in \mathbb{N}\setminus\{0\}} \frac{1}{n}\,u = \sup_{n \in \mathbb{N}\setminus\{0\}} \Big(-\frac{1}{n}\,u\Big) \leqslant w.$$

We obtain $0 \leqslant w$, and hence $0 \leqslant U(\kappa)$. Let $V \ni z \leqslant U(\kappa)$. Then $z \leqslant \frac{1}{n}u$ for all $n \in \mathbb{N} \setminus \{0\}$. As V is Archimedean, we get $z \leqslant 0$. Thus, $\inf_V U(\kappa) = 0$, what is required. $\rhd$

References

1. *Emel'yanov E. Yu.* Infinitesimals in ordered vector spaces // Vladikavkaz. Mat. Zh.—2013.—Vol. 15, № 1.—P. 18–22.

ИСПРАВЛЕНИЯ К СТАТЬЕ: «БЕСКОНЕЧНО МАЛЫЕ В УПОРЯДОЧЕННЫХ ВЕКТОРНЫХ ПРОСТРАНСТВАХ»

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