DIFFERENTIAL GEOMETRY AND APPLICATIONS
Proc. Conf., Aug. 28 – Sept. 1, 1995, Brno, Czech Republic
Masaryk University, Brno 1996, 231–238

ON FOUR-DIMENSIONAL CONFORMAL STRUCTURES

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ABSTRACT. The conformal structures $CO(4,0), CO(1,3)$ and $CO(2,2)$ are studied on a real manifold M, $\dim M = 4$. On M isotropic fiber bundles E_α and E_β are constructed. These bundles are real for the $CO(2,2)$-structure, and they satisfy the condition $\overline{E}_\alpha = E_\beta$ for the $CO(1,3)$-structure, and the conditions $\overline{E}_\alpha = E_\alpha$, $\overline{E}_\beta = E_\beta$ for the $CO(4)$-structure. The tensor C of conformal curvature splits into two subtensors C_α and C_β which are the curvature tensors of the bundles E_α and E_β, respectively. These subtensors satisfy the same conditions as the bundles E_α and E_β. Conformally semiflat and flat structures and their geometrical characteristics are studied. The principal 2-directions are defined, and conditions for their integrability are obtained. These investigations for the $CO(1,3)$-structure are connected with Petrov's classification of Einstein's spaces. Although the paper is of a servey nature, it also contains some new results.

1. Four-dimensional conformal structures play an important role in general relativity. Space-time in general relativity is a four-dimensional pseudo-Riemannian manifold of signature $(1,3)$. Many features of general relativity are of a conformally invariant nature. This is a reason that the study of pseudoconformal structures of signature $(1,3)$ is of special interest. Along with this kind of conformal structures, on a real four-dimensional manifold M, one also can consider conformal structures of signatures $(4,0)$ and $(2,2)$. By means of complexification of the manifold M, all these structures can be reduced to one of them, for example, to the structure of signature $(4,0)$ or $(2,2)$. This was done in many investigations (see, for example, [AHS 78], [Gi 82], [P 67], and [P 77]).

Unlike the previous investigations, in the present paper, we consider four-dimensional conformal structures on a real manifold M and study their common properties and the differences between them. Moreover, we also apply complexification not of the manifold M itself but only of its tangent spaces $T_x(M)$, and consider in these spaces coordinate transformations preserving the real part of these tangent spaces.

1991 *Mathematics Subject Classification.* 53A30.
Key words and phrases. Conformal structure, isotropic fiber bundle, tensor of conformal curvature, principal distributions, completely isotropic submanifolds, Petrov's classification.
This paper is in final form and no version of it will be submitted for publication elsewhere.

Some of the problems discussed in the current paper were considered earlier in more detail in the papers [A 83] and [AK 93]. In the paper [AZ 95], the apparatus constructed in the current paper is applied to the study of conformal structures of the Schwarzschild metric and the Kerr metric (see, for example, [Ch 83]) arising in general relativity.

2. Let M be a real differentiable manifold of dimension n endowed with a conformal structure of signature (p, q), $p + q = n$, which is the class $\{g\}$ of conformally equivalent Riemannian metrics g of signature (p, q). We will denote such a conformal structure by $CO(p, q)$ (see, for example, [K 72]).

A $CO(p, q)$-structure is a G-structure with the structure group $G = O(p, q) \times H$ where $SO(p, q)$ is the special pseudoorthogonal group of signature (p, q) and $H = R \cdot \mathrm{Id}$ is the group of homotheties.

Let $CT_x = T_x(M) \otimes C$ be the complexified tangent space $T_x(M)$ to the manifold M at a point x. In this space we will allow only complex linear transformations which preserve the real subspace T_x of CT_x and the symmetry with respect to $T_x(M)$. Denote by C_x the isotropic cone of the $CO(p, q)$-structure at the point x, $C_x = \{\xi \in CT_x | g(\xi, \xi) = 0\}$.

Let $\{e_i\}$, $i = 1, \ldots, n$, be a vectorial frame in the space $T_x(M)$ or CT_x, and $\{\omega^i\}$ be the coframe such that $\omega^i(e_j) = \delta^i_j$. In the space CT_x, the family of these frames must be invariant with respect to admissible transformations of this space.

3. For $n = 4$, from real point of view, there are three different types of conformal structures:

(i): The structure $CO(4, 0) = CO(4)$, for which by means of a real transformation, its fundamental form g can be reduced to the form:

$$g = (\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2 + (\omega^4)^2. \tag{1}$$

The isotropic cones C_x of this structure are pure imaginary. The $CO(4)$-structure is called *proper conformal.*

(ii): The $CO(1, 3)$-structure whose fundamental form g can be reduced to the form:

$$g = (\omega^1)^2 - (\omega^2)^2 - (\omega^3)^2 - (\omega^4)^2. \tag{2}$$

The isotropic cones C_x of this structure are real and have real rectilinear generators. The $CO(1, 3)$-structure is called a *structure of Lorentzian type.*

(iii): The $CO(2, 2)$-structure whose form g can be reduced to the form:

$$g = (\omega^1)^2 - (\omega^2)^2 + (\omega^3)^2 - (\omega^4)^2. \tag{3}$$

The isotropic cones C_x of this structure are real and have real two-dimensional plane generators. The $CO(2, 2)$-structure is called a *structure of hyperbolic type.*

By means of admissible transformations of coordinates in the space $CT_x(M)$, the quadratic forms (1), (2) and (3) can be reduced to the same form:

$$g = 2(\omega^1\omega^4 - \omega^2\omega^3), \tag{4}$$

The coordinates, with respect to which the form g has expression (4), are called *isotropic*.

For the $CO(2,2)$-structure, this transformation has the form:

$$\begin{aligned} &\omega^1+\omega^4 \to \sqrt{2}\,\omega^1, \quad \omega^2+\omega^3 \to \sqrt{2}\omega^2, \\ &\omega^1-\omega^4 \to \sqrt{2}\,\omega^4, \quad \omega^2-\omega^3 \to \sqrt{2}\,\omega^3 \end{aligned} \tag{5}$$

and is real. The isotropic coordinates ω^i, $i=1,2,3,4$, are real too.

For the $CO(1,3)$-structure, this transformation has the form

$$\begin{aligned} &\omega^1+\omega^4 \to \sqrt{2}\omega^1, \quad \omega^2+i\omega^3 \to \sqrt{2}\omega^2, \\ &\omega^1-\omega^4 \to \sqrt{2}\omega^4, \quad \omega^2-i\omega^3 \to \sqrt{2}\omega^3 \end{aligned} \tag{6}$$

and is complex. Moreover, the isotropic coordinates ω^i satisfy the conditions

$$\overline{\omega}^1=\omega^1, \quad \overline{\omega}^4=\omega^4, \quad \overline{\omega}^2=\omega^3. \tag{7}$$

For the $CO(4)$-structure, this transformation has the form

$$\begin{aligned} &\omega^1+i\omega^4 \to \sqrt{2}\omega^1, \quad \omega^2+i\omega^3 \to \sqrt{2}\omega^2, \\ &\omega^1-i\omega^4 \to \sqrt{2}\omega^4, \quad \omega^2-i\omega^3 \to -\sqrt{2}\omega^3 \end{aligned} \tag{8}$$

and is also complex. In this case, the isotropic coordinates ω^i satisfy the conditions

$$\omega^4=\overline{\omega}^1, \quad \omega^3=-\overline{\omega}^2, \tag{9}$$

With respect to the new basis, the equation $g=0$ of the isotropic cone C_x can be written in two different forms:

$$\frac{\omega^1}{\omega^3}=\frac{\omega^2}{\omega^4}=-\lambda \quad \text{and} \quad \frac{\omega^1}{\omega^2}=\frac{\omega^3}{\omega^4}=-\mu, \tag{10}$$

i.e., in the form of two systems of equations:

$$\omega^1+\lambda\omega^3=0, \;\; \omega^2+\lambda\omega^4=0 \tag{11}$$

and

$$\omega^1+\mu\omega^2=0, \;\; \omega^3+\mu\omega^4=0. \tag{12}$$

The systems (11) and (12) determine two-dimensional plane generators $\alpha(\lambda)$ and $\beta(\mu)$ of the isotropic cone C_x of the four-dimensional conformal structure which are called α-*planes* and β-*planes*, respectively (see [PR 86]).

Since for the $CO(2,2)$-structure the isotropic coordinates ω^i are real, the parameters λ and μ in equations (11) and (12) are real nonhomogeneous coordinates on the projective lines RP_α and RP_β, respectively. The α- and β-generators of the cones C_x are also real. On the manifold M, they form the two fiber bundles $E_\alpha=(M,RP_\alpha)$ and $E_\beta=(M,RP_\beta)$, respectively, which are called the *isotropic fiber bundles*.

The structure group G of the $CO(2,2)$-structure decomposes into the product: $G=SL(2)\times SL(2)\times H$.

For the $CO(1,3)$- and $CO(4)$-structures, the isotropic coordinates ω^i are complex. Hence the parameters λ and μ in equations (11) and (12) should be considered as complex nonhomogeneous coordinates on the projective lines CP_α and CP_β, respectively. For these structures, the α- and β-generators of the cones C_x are also complex and form the fiber bundles $E_\alpha = (M, CP_\alpha)$ and $E_\beta = (M, CP_\beta)$ on the manifold M.

Moreover, for the $CO(1,3)$-structure, by equations (7), the α- and β-planes are related by the condition $\overline{\alpha(\lambda)} = \beta(\overline{\lambda})$. Thus, *the isotropic bundles E_α and E_β are complex conjugates: $\overline{E}_\alpha = E_\beta$*. The generators $\alpha(\lambda)$ and $\beta(\overline{\lambda})$ of the cone C_x have the common rectilinear generator defined by the vector $\xi = \lambda\overline{\lambda}e_1 - \lambda e_2 - \overline{\lambda}e_3 + e_4$. The structure group G of the $CO(1,3)$-structure admits two isomorphic representations on the fiber bundles E_α and E_β: $G \cong SL(2,C) \times H \cong \overline{SL(2,C)} \times H$, where the groups $SL(2,C)$ and $\overline{SL(2,C)}$ act concordantly on the fiber bundles E_α and E_β.

For the $CO(4)$-structure, by equations (9), the α-planes and β-planes are related by the conditions $\overline{\alpha(\lambda)} = \alpha(-\frac{1}{\overline{\lambda}})$ and $\overline{\beta(\mu)} = \beta(-\frac{1}{\overline{\mu}})$. Thus, the fiber bundles E_α and E_β are self-conjugate: $\overline{E_\alpha} = E_\alpha$, $\overline{E_\beta} = E_\beta$, and the structure group G of this structure decomposes into the product $G = SU(2) \times SU(2) \times H$.

Since on the fiber bundle E_α, an α-plane is determined by a point x and a value of the parameter λ, sometimes we will denote it by (x,λ). By a similar reason, we will denote a β-plane by (x,μ).

4. Denote by C the *tensor of conformal curvature* of a four-dimensional conformal structure which is called also the Weyl tensor (see, for example, [SS 38] and [A 83]), and by C_α and C_β the curvature tensors of the isotropic fiber bundles E_α and E_β. The vanishing of the tensor C characterizes the *conformally flat structures*, and the vanishing of one of the tensors C_α and C_β characterizes the *conformally semiflat structures*.

A Riemannian manifold endowed with the conformally flat structure can be mapped into the conformal space C^4 or the pseudoconformal spaces C^4_1 and C^4_2 for which the quadratic form g has signature (4, 0), (1, 3) and (2, 2), respectively. Note that the conformal space C^4_q is a compactified Euclidean space R^4_q, i.e. it is R^4_q enlarged by the point at infinity and the isotropic cone C_∞ with vertex at this point: $C^4_q = R^4_q \cup C_\infty$.

The connection between the tensors C, C_α and C_β can be described as follows.

For the $CO(2,2)$-structure, the tensor C decomposes into the sum of the tensors C_α and C_β: $C = C_\alpha \dot{+} C_\beta$, and each of the tensors C_α and C_β has five independent components:

$$C_\alpha = \{a_0, a_1, a_2, a_3, a_4\}, \;\; C_\beta = \{b_0, b_1, b_2, b_3, b_4\}$$

(see [A 83]).

For the $CO(1,3)$- and $CO(4)$-structures, the tensors C_α and C_β have complex coordinates a_u and $b_u, u = 0,1,2,3,4$. Moreover, the relation $\overline{E}_\alpha = E_\beta$ between the isotropic fiber bundles of the $CO(1,3)$-structure implies a similar relation $\overline{C}_\alpha = C_\beta$ for their curvature tensors.

Similarly the relations $\overline{E}_\alpha = E_\alpha$ and $\overline{E}_\beta = E_\beta$ between the isotropic fiber bundles of the $CO(4)$-structure imply similar relations $\overline{C}_\alpha = C_\alpha$ and $\overline{C}_\beta = C_\beta$ for their curvature tensors. One can prove that these relations can be written in the following more detailed form:

$$\begin{aligned} &\overline{a}_0 = a_4, \quad \overline{a}_1 = -a_3, \quad \overline{a}_2 = a_2, \\ &\overline{b}_0 = b_4, \quad \overline{b}_1 = -b_3, \quad \overline{b}_2 = b_2. \end{aligned} \tag{13}$$

It follows from these considerations that *only the* $CO(2,2)$*- and* $CO(4)$*-structures can be conformally semiflat, and the* $CO(1,3)$*-structure cannot be conformally semiflat without being conformally flat.* It is not difficult to show that the conformally semiflat $CO(4)$-structures coincide with the self-dual and anti-self-dual structures considered in [AHS 78].

5. Denote by $V_x^2 = \{p\}$ the bivector space formed by the bivectors $p = \xi \wedge \eta$ of the tangent space $T_x(M)$ to a manifold M endowed with $CO(p,q)$-structure. Since here $\xi = \xi^i e_i$ and $\eta = \eta^i e_i$ are vectors of the space T_x, we have $p = p^{ij} e_i \wedge e_j$ where $p^{ij} = \frac{1}{2}(\xi^i \eta^j - \xi^j \eta^i)$, $i, j = 1, 2, \ldots, n$. The quadratic form

$$C(p) = C_{ijkl} p^{ij} p^{kl},$$

where C_{ijkl} are components of the Weyl tensor in an arbitrary coordinate system is called the *relative conformal curvature* of the bivector p.

Consider the $CO(2,2)$-structure, and let $p_\lambda = \xi_\lambda \wedge \eta_\lambda$ be an isotropic bivector belonging to the fiber bundle E_α. The coordinates of the vectors ξ_λ and η_λ in the isotropic frame satisfy equations (11), and this implies that

$$\xi_\lambda = e_3 - \lambda e_1, \quad \eta_\lambda = e_4 - \lambda e_2.$$

In [A 83] it was proved that the relative conformal curvature of the bivector p_λ is expressed by the formula

$$\frac{1}{4} C(p_\lambda) = a_0 \lambda^4 - 4a_1 \lambda^3 + 6a_2 \lambda^2 - 4a_3 \lambda + a_4 := C_\alpha(\lambda). \tag{14}$$

Similarly, the relative conformal curvature of the bivector $p_\mu = \xi_\mu \wedge \eta_\mu$ has the form

$$\frac{1}{4} C(p_\mu) = b_0 \mu^4 - 4b_1 \mu^3 + 6b_2 \mu^2 - 4b_3 \mu + b_4 := C_\beta(\mu). \tag{15}$$

The coefficients of the polynomials (14) and (15) are components of the tensors C_α and C_β which are real for the $CO(2,2)$-structure.

The isotropic bivector p_λ or p_μ for which relative conformal curvature vanishes, i.e. for which $C_\alpha(\lambda) = 0$ or $C_\beta(\mu) = 0$, is called the *principal isotropic bivector.*

It follows from formulas (14) and (15) that *the* $CO(2,2)$*-structure is* α*-semiflat (* β*-semiflat) if and only if all isotropic bivectors belonging to the fiber bundle* E_α *(respectively, to* E_α*) are principal. This structure is flat if and only if all isotropic bivectors are principal.*

A two-dimensional submanifold V of the manifold M endowed with a pseudo-conformal $CO(2,2)$-structure is called *completely isotropic* if for any $x \in V$, the tangent 2-plane $T_x(V)$ is isotropic. If $T_x(V) \subset E_\alpha$, then the submanifold is called *α-isotropic* and denoted by V_α; and if $T_x(V) \subset E_\beta$, then $V = V_\beta$ is a *β-isotropic* submanifold.

The submanifolds V_α are determined on M by the system of equations (11). Exterior differentiation of equations (11) leads to the equations

$$\theta_\lambda \wedge \omega^3 = 0, \quad \theta_\lambda \wedge \omega^4 = 0, \tag{16}$$

where

$$\theta_\lambda := d\lambda + \lambda(\theta_1^1 + \theta_2^2) - \theta_3^1 + \lambda^2 \theta_1^3,$$

and $\theta_1^1 + \theta_2^2$, θ_3^1, θ_1^3 are invariant forms of the group $GL(2)$ acting in the fiber bundle E_α. Since the forms ω^3 and ω^4 are linearly independent on the submanifold V_α, it follows from equation (16) that

$$\theta_\lambda = 0. \tag{17}$$

By taking the exterior derivative of this equation, we obtain $C_\alpha(\lambda) = 0$.
Thus, *all tangent 2-planes $T_x(V_\alpha)$ to an α-isotropic submanifold are α-principal.*

In exactly the same way, we can prove that *all tangent 2-planes $T_x(V_\beta)$ to a β-isotropic submanifold are β-principal.*

A $CO(2,2)$-structure is called *α-semiintegrable* if for any α-plane (x, α), there exists a submanifold V_α such that $T_x(V_\alpha) = (x, \alpha)$. In this case, the system of equations (11) and (17) is completely integrable and defines a three-parameter family of completely isotropic submanifolds V_α on M.

The following theorem is valid: *A $CO(2,2)$-structure is α-semiintegrable if and only if it is α-semiflat.*

In a similar way, one can define β-semiintegrable $CO(2,2)$-structures and prove that the class of such structures coincides with the class of β-semiflat structures. Finally, *a $CO(2,2)$-structure is simultaneously α- and β-semiintegrable if and only if it is conformally flat.*

Suppose that the $CO(2,2)$-structure is not conformally semiflat. Then each of the equations

$$C_\alpha(\lambda) = 0 \tag{18}$$

and

$$C_\beta(\mu) = 0 \tag{19}$$

has four roots if we count each of these roots as many times as its multiplicity. Denote these roots by λ_p and β_p, $p = 1, 2, 3, 4$, respectively. Each of these roots determines an isotropic distribution which is called *principal* and denoted by $\Delta_p(\alpha)$ and $\Delta_p(\alpha)$, respectively.

A pseudoconformal $CO(2,2)$-structure is called *α-semirecurrent* if $\nabla C_\alpha = \sigma_1 C$; it is called *$\beta$-semirecurrent* if $\nabla C_\beta = \sigma_2 C$; and it is called *recurrent* if $\nabla C = \sigma C$. Here ∇ is symbol of the covariant differential with respect to the invariant conformal connection defined on M, and σ, σ_1 and σ_2 are 1-forms (cf. [AM 67]).

The following theorem is valid.

Theorem ([A 83])
a) *If the $CO(2,2)$-structure is α-semirecurrent (respectively, β-semirecurrent), then all principal distributions $\Delta_p(\alpha)$ (respectively, $\Delta_p(\beta)$) are integrable. If this structure is recurrent, then all principal distributions are integrable.*
b) *If λ_p (respectively, μ_p) is a multiple root of equation* (18) *(respectively,* (19)*), then the principal distribution $\Delta_p(\alpha)$ (respectively, $\Delta_p(\beta)$) determined by this root is integrable.*

The theory developed in this section for the $CO(2,2)$-structure can be immediately extended to the $CO(1,3)$- and $CO(4)$-structures. In addition, a few more results following from the properties of their tensors C_α and C_β can be added.

Since for the $CO(1,3)$-structure, we have $\overline{C}_\alpha = C_\beta$, equations (18) and (19) have complex conjugate roots: $\mu_p = \overline{\lambda}_p$. Hence the principal α- and β-bivectors are also complex conjugate, and for the principal distributions, the relation $\overline{\Delta_p(\beta)} = \Delta_p(\alpha)$ holds. Thus, the distribution $\Delta_p(\beta)$ is integrable if and only if the distribution $\Delta_p(\alpha)$ is integrable. The intersection of the principal bivectors α_p and β_p determines a real vector field $\xi_p = \alpha_p \wedge \beta_p$, whose integral curves are *isotropic geodesics with respect to any Riemannian connection compatible with the $CO(1,3)$-structure.* In the general case, *a manifold M endowed with the $CO(1,3)$-structure carries four isotropic geodesic congruences.*

For the $CO(4)$-structure, the components of the tensor C_α satisfy the equation

$$C_\alpha(\lambda) = a_0\lambda^4 - 4a_1\lambda^3 + 6a_2\lambda^2 + 4\overline{a}_1\lambda + \overline{a}_0 = 0,$$

where $\overline{a}_2 = a_2$. Thus, the roots of this equation are connected by the relations $\lambda_4 = -(\overline{\lambda}_1)^{-1}$ and $\lambda_3 = -(\overline{\lambda}_2)^{-1}$. It follows that the roots λ_1 and λ_4 as well as λ_2 and λ_3 cannot coincide. Furthermore, *if $\lambda_1 = \lambda_2$, then $\lambda_3 = \lambda_4$*. Thus, *the above equation has either four distict roots or two pairs of double roots; in the later case, the isotropic fiber bundle E_α carries two double principal α-distributions.* However, since these distributions are complex, they do not define foliations on the real manifold M.

6. Space-time in general relativity is a four-dimensional pseudo-Riemannian manifold of signature (1, 3) of special type. Such manifolds are called *Einstein's spaces* (see, for example, [Ch 83]) and are defined by the equation

$$R^i_{jki} = \alpha g_{jk} + \beta T_{jk},$$

where R^i_{jkl} is the tensor of conformal curvature, and T_{jk} is the energy-momentum tensor.

Many features of general relativity are related to the conformal structure of space-time since its light cones coincide with the isotropic cones of the $CO(1,3)$-structure defined by its Riemannian metric. In particular, Petrov's classification of Einstein's spaces is of conformally invariant nature. This classification is based on the structure of roots of equations (18) and (19). Since for the $CO(1,3)$-structure, these equations are complex conjugate, we will consider only equation (18).

In terms of the present paper, Petrov's classification is represented in the form of the following table:

Petrov's type	Roots of the equation $C_\alpha(\lambda) = 0$	Characterization of principal distributions	Characterization of isotropic geodesic congruences
I	$\lambda_p \neq \lambda_q,\ p \neq q,$ $p, q = 1, 2, 3, 4$	4 different of general type	4 simple
II	$\lambda_1 = \lambda_2 \neq \lambda_3, \lambda_4;$ $\lambda_3 \neq \lambda_4$	1 double and 2 of general type	1 double and 2 simple
D	$\lambda_1 = \lambda_2 \neq \lambda_3 = \lambda_4$	2 double	2 double
III	$\lambda_1 = \lambda_2 = \lambda_3 \neq \lambda_4$	1 triple and 1 of general type	1 triple and 1 simple
N	$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$	1 quadruple	1 quadruple

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