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Reference Levels in School Mathematics in Europe

Short unofficial summary

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REFERENCE LEVELS IN SCHOOL MATHEMATICS IN EUROPE

This short unofficial summary seeks to convey

- A sense of the variety to be found in different countries;
- Some of the most striking similarities and differences between countries;
- Some interesting examples.

For precise information readers must consult the national reports (which contain many fascinating details), the official report, and the more complete documentation from national and regional Ministries of Education - much of which has been collected in the project's documentation centre in Besançon.

This summary deliberatee tries to unify and simplify what is in fact a very complicated picture; it is therefore intended only as a starting point, providing beginners with a broad general picture: it should definitely *not- be used as a formal reference. In particular, the summary errs on the side of trying to invent a common language (of ages and approximate "grades", and of school phases) which does not conform exactly to any of the systems described, but which is only meant to be correct "to a first approximation".

In reality, the systems being described differ widely in

- the age at which compulsory schooling begins and ends;
- in the way the time spent in school is broken up into "cycles";
- in the place (if any) given to vocational schools or programmes;
- in the way different types of school are classified;

and

- in the social and political traditions which underlie their individual characteristics.

The true picture is made even more complicated by the fact that a surprising number of the educational systems surveyed are currently in a state of transition.

1. General description of the mathematics teaching context

1.1 The school system

The age at which formal schooling begins varies considerably - from 4/5 in England, to 7 in Finland and Russia. The starting age in any particular country depends on historical traditions, social conditions and political decisions. However, it is striking that those countries which start earlier do not appear to cover more mathematics by the end of compulsory schooling: if anything, the opposite may even be the case.

The length of compulsory schooling also varies - from 9 years in Italy, Finland, Greece and Russia, to 11/12 years in England (ending at age 16), and 12 years in Belgium (where a later start and parallel structures for academic and vocational provision make it possible for compulsory schooling to continue until age 18).

The types of school also vary considerably from system to system.

Almost all countries have an initial "primary" phase - ending somewhere between age 10 and age 13. This may be subdivided into separate "cycles" (e.g. the 2+2+2 system in Spain).

The primary phase is often followed by a "lower secondary" phase (ending somewhere between age 14 and age 16), after which pupils move on to an upper secondary phase. Where the upper secondary phase begins early, it is part of the compulsory system; where it begins around age 16, it is usually not compulsory and is often split into separate "academic" and (possibly multiple) "vocational/technical" programmes, which run in parallel - often in separate institutions.

Sometimes the first two phases are combined into a single "basic" programme (as in Russia, where it lasts 9 years).

At the "secondary" phase (which may begin at any age from 10 to 14) schools may be labelled and distinguished in different ways. The easiest systems to understand are those which are uniform and fully comprehensive (as in Finland and Sweden). Here one finds a strong emphasis on "equality of opportunity". For us this may be interpreted as implying that the mathematics course should be accessible to all pupils; yet this does not lead to a single rigid programme - for the flexibility built into these systems means that there is considerable scope for local variation and initiative. For example, in Sweden teachers and schools are responsible for what is taught, and for assessment; the National Curriculum only provides outline goals for the end of Grades 5 and 9 (age 16), and states criteria to guide local assessment. Examinations are provided centrally, but administered and marked locally. In general, the fact that the programme has to be taught to all pupils means that goals of the mathematics curriculum up to age 16 have to be "realistic": hence items which are judge to be technically demanding tend to appear first at upper secondary level (post 16), which can distort the curriculum. In some respects the Spanish structure is similar: the national curriculum provides general goals, which are then interpreted at the local and regional level; and formal assessment is the responsibility of individual schools.

Russian schools are also mostly of a single type up to age 16. However, in contrast to the Swedish model, Russia interprets its tradition of "equality of opportunity" by offering a *technically demanding- curriculum to almost all pupils. This curriculum is effectively determined by centrally set and administered examinations.

Other countries (such as Germany, Italy, Hungary, Belgium) offer distinct curricula in separate institutions above some age between 10 and 14. The variations on this theme are many: there may be just two types of institution (roughly "university-oriented" and "employment-oriented", as in Belgium); there may be essentially three types (as in parts of Germany); or there may be more types (as in Italy). In essence however, the types of schools distinguish between "academic" and "technical/vocational" - though there may be more than one type of "academic" secondary school and many different sub-types of "vocational/technical" school (as in Italy and Germany). In countries such as Russia, Sweden, France and Finland, the distinction between "academic" and "technical/vocational" is made only at upper secondary level (after the age of 15/16).

In some countries (such as England and parts of Germany) apparently incompatible systems - such as comprehensive and selective - can exist side by side. One may even find (e.g. in England) school types that do not conform to any single structure, but rather represent a patchwork of successive political initiatives.

Countries vary considerably in the extent to which the curriculum and its assessment are centrally controlled. There is no simple pattern. Countries with what looks like a strongly principled and uniform system (such as Finland and Sweden with comprehensive secondary schools for all, or Belgium with just two types of secondary school) may offer schools and teachers a degree of freedom to interpret the curriculum and either to devise and operate, or to administer and mark, the formal assessment system. In Hungary only 50-70% of the secondary curriculum is centrally determined; thus the national curriculum provides a partial framework leaving schools flexibility to develop or to supplement its themes. Other countries are strongly driven by a central curriculum (as in France, Greece, Italy and England), and/or by centrally monitored and administered assessment (as in Greece and Russia). England and France operate rather different systems of regular, centrally administered assessment. [In general it would be interesting to know more about the variation between countries in the character, style and timing of centrally administered assessment.] The English system appears to be unique in that the results of the recently introduced national tests at ages 7, 11, 14, 16 are made public in the form of "league tables" - ranking schools from the "best" to the "worst". The intention is to identify weak schools and so to force them to change. While this intention has been partially successful, the overall effect in mathematics has been to constrain even the best teachers to focus on short-term test results at the expense of laying strong foundations for long-term development.

1.2 Place and importance of mathematics in the curriculum

The importance of mathematics in the curriculum is most easily characterised by the amount of time which schools actually spend, or are officially obliged to devote to, teaching mathematics. This varies considerably. In some countries the amount of time per week is specified for each phase and each school type or programme. In others, the *total- amount of time is specified within a single phase. Elsewhere the approximate percentage is either specified, or understood, for each phase. Or there may be no official requirements, so that one can only report on "typical" time allocations.

Whether it is officially required, or depends on tradition and practice, the time for mathematics during the primary phase is generally around 20%, while in lower secondary it varies between 10% (8%?) and 15%. For example, in Grades 7-9 (ages 14-16) in Russia, mathematics is perceived as being very important, with 5 periods a week being devoted to the two main strands of Geometry and Algebra; there is no probability or statistics at this level. In some countries (e.g. England) the time devoted to mathematics has been substantially reduced in recent years (by as much as 20% at secondary level). Such reductions are often the unintended result of additional demands being imposed on schools, rather than reflecting considered policy.

The recognised importance of mathematics means that it is usually compulsory in some form up to age 16; indeed most countries also require some study of mathematics in the upper secondary phase (to age 18). Exceptions vary from England (where only 15% or so continue with mathematics, though many others choose to retake their age 16 mathematics exam), to Finland (where all students are supposed to take a mathematics course, but they can avoid taking mathematics as part of their "Matura" examination), and Hungary (where all students completing their "Matura" at age 18 have to include some mathematics).

The English structure is unusual in that it has no year-by-year curriculum. Hence, there is no notion of what is meant by "Grade 7 mathematics". Rather, the curriculum is structured in the form of a single ladder, consisting of nine "levels", up which all pupils are expected to climb "at their own pace". Thus the official curriculum *accentuates- differences from an early age; nevertheless, the English system does not provide for different constituencies by offering a choice of "academic" or "vocational/technical" institutions at some point between age 10 and 14. Instead, the dilemma is resolved in an ad hoc way by recognising that many pupils will not achieve the goals outlined in the official curriculum for age 14 and age 16, and by then providing three different levels of mathematics exam at age 16.

2. Main mathematics objectives

It is in practise very difficult to identify the "main mathematics objectives" of each school system. The reason is tellingly expressed in the Russian national report. There we read:

"The national curriculum in mathematics indicates the following objectives of learning mathematics in school:

I - mastering specific mathematical skills [...];

II - developing students intellectually [...];

III - forming the conception of mathematics as a source of tools for describing and studying the real world;

IV - forming a conception of mathematics as part of human culture.

This list is in good agreement with the [official] structure. [...] However [...] most effort is directed toward achieving the first objective. [...;] the other three objectives remain declarations, and are achieved only insofar as they arise in the process of acquiring skills."

In some systems (e.g. Hungary), the objectives include interesting enlightened statements which appear to be well-matched to the highly technical nature of the curriculum. In other systems (e.g. various German Lender) one may find reference to wider abilities, but official discussions and documents take it for granted that formal requirements should focus mainly on things which can be seen (such as the acquisition of techniques) rather than more elusive "general abilities".

In some countries (such as Poland) the objectives are expressed mainly in terms of broad - yet still mathematical - notions (concepts, use, problem solving, spatial ability, logic, etc). In many other countries national curricula begin with lists of "soft" objectives (concerning such themes as communication, groupwork, etc.) which bear little relation to what is actually taught in ordinary classrooms - or perhaps what, in reality, one can expect to be taught. The meaning of these lists often remains unclear, partly because they are rarely formulated as eloquently as the Russian (and Hungarian) statements.

However, one sometimes finds hints of a clearer conceptual structure: for example,

(i) in some countries (e.g. Belgium) there is a clear recognition that "techniques/skills" and "broader understandings" (such as abilities to analyse problems and to use mathematical language correctly and effectively) represent complementary ways of thinking about "objectives"; and

(ii) elsewhere (e.g. in some German Lender) one finds a clear distinction between what is expected of different kinds of students - distinguishing between certain objectives for those in Gymnasium and for those in Hauptschulen.

Which raises the question as to why so many systems include these lists of "softer" objectives, when their ordinary teachers and examiners persist in focusing on more mundane, and more achievable, matters. Indeed, there are many who teach in universities (e.g. in England and Germany) who have suggested that school mathematics in their countries would be more effective if schools were encouraged to concentrate on those "enlightened" objectives which help pupils to experience mathematics as a rational enterprise, and to acquire greater technical fluency and the flexibility of mind required to use these techniques effectively. In such a setting, lists of "higher" objectives might be restricted to those which can be interpreted, addressed explicitly, and assessed as part of such a programme.

The general issue here is the extent to which mathematics should be used as a springboard for the inculcation of what many claim to be important "generic skills" (such as "communication", "problem-solving", and "teamwork"), or whether this shift of emphasis from "lower" to "higher" goals tends to distract attention from what can actually be achieved, and so reduces the likelihood of a programme achieving more fundamentally important goals.

In considering such questions, one should bear in mind that there are countries where broader pedagogical and social objectives are interwoven with strictly mathematical objectives in a more coherent manner. For example, several countries explicitly intend to produce effective "realistic problem solvers" (e.g. Belgium), and genuinely try to devise curricula which might achieve such goals. Moreover, the political and social thinking behind the comprehensive systems of Sweden and Finland indicate a commitment to ensuring that all students in lower secondary school achieve confidence in using the mathematics they learn, and an understanding of the wider significance and applicability of mathematics in society, rather than merely mastering techniques. This "civic" vision may be especially important for those pupils whose eventual mastery of mere technique is likely to be limited.

Thus in some countries, these lists of broader objectives are seen as an essential part of the system; in others, they are apparently included mainly to satisfy a powerful educational lobby, and remain largely at the level of wishful thinking. [An interesting case study would be to examine the varied nature of stated objectives which declare the need for a strong link between school mathematics and modern technology, and to examine the impact which such objectives have on what is actually learned by the age of 16.]

3 Basic contents

The basic contents covered in each system cannot be easily summarised. For details, we refer readers to the national reports and national curriculum documents. However, we should stress that, as the examples given in section 4 below illustrate, the occurrence of similar words in the curricula of different countries should *never- be taken to indicate that the systems teach more-or-less the same things. Rather one has to read between the lines (and consult textbooks and examination papers) in order to assess the intended style and spirit, and the depth of understanding which is routinely achieved - since, on closer inspection, these may turn out to be worlds apart.

One distinction which should perhaps be mentioned is that between

- countries who adopt a tightly focused curriculum up to age 16, which material they then seek to cover to some depth,

and

- countries whose curriculum has expanded (e.g. to include work on data handling, or a commitment to use technology) in such a way as to make coverage-in-depth impossible.

Neither group is homogeneous. The first group includes both the technically demanding, but tightly focused curriculum in Russia and the more "democratic" curricula in Sweden and Finland. The latter group includes countries like England.

To encourage the reader to look more closely at the national reports we offer here, by way of illustration, some brief comments on the thrust and style of Geometry teaching for pupils aged 13-16. In many of the countries included in this survey one finds a rich geometrical diet for (almost) all pupils - focused strongly on work in 2D (two dimensions).

Some countries go out of their way to ensure that the curriculum includes substantial work on geometry in three dimensions (e.g. Poland). However, along with this emphasis one generally finds a recognition that successful work in 3D depends on choosing suitable 2D sections of the given 3D configuration in order to apply 2D techniques (e.g. Pythagoras). Thus, success depends on pupils having already mastered the corresponding 2D techniques, and then being helped to develop the ability to identify the relevant 2D sections in 3D configurations in order to apply the relevant 2D techniques.

Similarly, while some official programmes include mention of "transformations", the basic tools for their analysis are generally the traditional ones using triangles, parallelograms, etc., and (modified) euclidean notions of congruence and similarity. Thus, the basic properties of isometries are likely to be analysed initially via congruence (as in Hungary, Italy, Greece, or Belgium).

Analogous comments apply to the use of computer software in geometry teaching. Namely, where its use is encouraged (as in France, Greece and Italy), this positive encouragement presupposes, builds on, and is used to develop and extend relatively classical geometrical skills.

We end this section with a selection of glimpses into the detail of particular national reports. The Italian curriculum is geometrically very rich; but it also includes a tradition of using word problems to help pupils develop the ability to use formal results and methods to solve problems. The Russian curriculum has an explicit section on "ruler and compass constructions" - exploiting this classical topic to help pupils achieve a hands-on feeling for geometrical configurations and an informal insight into the nature of a local deductive system. The distinctive nature of the Swedish curriculum is well illustrated in the section on geometry, where it merely refers to "properties of common geometrical objects" - making no mention of triangles, squares, or circles: the intention however is not to downplay the significance of triangles etc., since these "geometrical objects" are recognised as being central to the mathematics syllabus. Rather the formulation reflects the fact that it is the responsibility of schools to interpret the centrally formulated curriculum at local level.

4. Exemplary topics

We comment here only on 4.1, 4.2 and 4.4.

4.1 Quadratic equations

The general method for solving quadratic equations (with a proof based on completing the square) is covered by age 16 in many countries: e.g. Spain, Poland, Belgium, Hungary, Greece, Russia.

In Germany the full result (with proof via completing the square) is covered by pupils in the Gymnasium, while those attending a Hauptschule would usually meet only the simple case where no linear term is present. Similarly in Italy the general result is treated in the first or second year of secondary school for 50% of pupils, and one year later for the rest. In Finland and Sweden, simple special cases would be met by all pupils by age 16, but the general method is treated only later at upper secondary level. In England only pupils taking the "higher tier" at age 15-16 (~20%) will meet the general result.

In Spain, Greece, Poland and Italy parabolas and quadratic functions are treated before the general solution of quadratic equations. In some countries (e.g. Belgium) work on quadratic equations is linked to the analysis of general quadratic functions (linking the algebraic notion of a change of variables with the geometric idea of shifting the origin).

4.2 Pythagorean theorem

In many countries (e.g. Poland, Germany, Greece, Spain, Russia, Italy, Finland, Sweden) all pupils would meet the Pythagorean theorem by the age of 16. In Italy, some motivation is presented when the result is first met, but the topic is treated more thoroughly later in secondary school. In Germany, a full proof would be expected - and the converse would be stated - for those in a Gymnasium; those in a Hauptschule would meet the result without proof. In Finland and Sweden no proofs would normally be given. In Spain, some visual image may be used to make the result plausible, but a proof is unlikely to be given. In Russia, the proof is compulsory and the converse would also be treated.

In Belgium the result would be presented with proof to almost all pupils in the academic-oriented lower secondary school, and without proof to some in the technical/vocational school; thus roughly 50% of pupils meet the result by the age of 16. In England, around 40% of pupils are expected to have met the result by age 16, but there is no official mention of the need for a proof, and none is usually given.

4.4 Word problems

In many countries (e.g. Poland, Russia, Italy, Finland, Sweden, Germany, Spain) word problems constitute a core topic from late primary school onwards. In Finland, Poland and Russia they are recognised as central from the very beginning of primary school.

The Belgian national report states that there is "no particular emphasis" on word problems; however, other information (such as the declared change of official focus towards "realistic problem solving") suggest that word problems may in fact be taken seriously, but without being given a special name or label.

Many of these countries devote time to word problems because they are perceived:

I - as a vehicle for training pupils to use the elementary mathematics they have learned, in a way that cultivates flexibility of thinking with simple material;

II - as offering a clear link between school mathematics and familiar real situations;

III - as providing a rich source of settings in which pupils learn to interpret the world around them in mathematical terms (that is, word problems are seen to represent the simplest, and most common, form of "mathematical modelling"), and an accessible format for problems whose solution forces pupils to think and to exercise judgement (rather than merely applying some standard routine);

In all countries pupils (and teachers) find word problems demanding. But this is generally seen as indicating the need for careful teaching and lots of practice, rather than as a reason for them to be abandoned.

5. Other subjects of interest for mathematics education at age 16

Section 5 of the various national reports contains much interesting information. But it is information that is difficult to summarise in a coherent way. We therefore restrict attention in this informal summary to sections 5.1 and 5.2.

5.1 Regional variations

We begin by grouping the different systems under three headings - though we shall soon see that things are not quite so simple.

(a) On the one hand, there are countries (such as France, Greece, Italy or England) where there is officially no scope for significant regional variation.

(b) At the opposite extreme lie countries like Germany whose constitution creates a federation of constituent "states" or "autonomous regions", with decisions about education being devolved to regions.

(c) In addition there are countries which may not be strictly federal, but which include distinct language, or cultural, communities. The distinct communities may nominally follow similar programmes, yet may be permitted to do so in a manner which is consistent with their own traditions (as in Belgium, Hungary, and even Finland - with its Swedish speaking minority).

However, in practice, even in those countries where there is no official scope for regional variation, the reality on the ground may tell a different story. Hence a number of such countries report marked - though scarcely official - differences between provision in urban and in rural areas. And the quality of local provision can be strongly influenced by local government decisions (not least because local government is often the teachers' employer, and is responsible for their working conditions and in-service opportunities).

On the other hand, where the constitution recognises that education is a regional responsibility one may find a nominal multiplicity of systems, most of which show striking similarities to each other.

In Spain this is partly because there is an agreed outline "national curriculum" which provides a basic framework for the 17 autonomous regions, and partly because mathematics is less subject to regional pressures than some other subjects.

In Germany each of the 16 Lender is responsible for its own curricula. Allowing for the existence of parallel "academic" and "technical/vocational" programmes, this gives rise to perhaps 40 distinct mathematics curricula at lower secondary level! Yet this variety is moderated by exchange and debate between the Lender, which tends to reduce the extent of the differences in *content- (though the school *systems- and "assessment modes- differ in interesting ways from Land to Land).

England has a centrally determined curriculum; but Scotland, Wales, and Northern Ireland are each responsible for their own curricula. And, unlike Germany, the four systems would appear to be becoming increasingly detached from each other.

In Russia the time for mathematic is centrally controlled. But only 50% of the curriculum is centrally determined, leaving room for local variation.

5.2 Implementation strategies

We end by briefly indicating something of the range of implementation strategies in the different countries.

Only Finland reported a political mechanism which might allow a systematic approach. Every four years there is a Government Development Plan which provides a clear opportunity to specify how planned initiatives are to be implemented and how these are to be funded.

The other responses in this section were all more or less inadequate or *ad hoc- - reflecting a shortage of money and of effective structures for implementation.

In Poland innovations are often spread via a "cascade" system - with "leaders" being trained first, and then sent to train ordinary teachers. In Russia, Greece and Poland the fact that all textbooks have to be centrally approved provides a powerful, if indirect, mechanism for controlling change. The same is true in Germany - though at the level of individual Laender. In Russia prospective textbooks are analysed before being tested in pilot schools: only those which prove effective in practice are then officially recommended. In Germany and Greece the whole implementation process operates "top-down", with the drafting of syllabi and the official approval of textbooks as the main motors of change.

In England routine implementation of policy is done by the separate agencies responsible

(i) for curriculum and assessment,

(ii) for initial teacher training, and

(iii) for school inspections;

however, the most striking recent implementation - the *National Numeracy Strategy- - has been controlled directly by the Ministry of Education. Similarly in Sweden there was a centrally driven and funded programme from 1986-1990 in response to Sweden's poor showing in the *Second International Mathematics Study*, which was deemed successful in the light of improved performance in TIMSS; however, in general most practical support is left to municipalities, who, as the teachers' employers, are responsible for in-service provision.